I. INTRODUCTION

One of the most direct methods$^1$ for measuring the magnitude of the $K^0\bar K^0$ mass difference is that of determining the time dependence of the strangeness of neutral $K$ mesons of known initial strangeness. Figure 1 shows in a phase diagram the relation of $K^0$, $K^0$, and the resulting $K^0$, $K^0$ amplitudes at $t=0$ (production of a $K^0$) and at a later time $t$. This leads to the time dependence of the $K^0$ intensity

$$I(t)=e^{-\lambda t}+\epsilon e^{-\lambda t}+2\epsilon e^{(\lambda+\lambda t)}\cos[(\Delta m^2/\hbar)t],$$

(1)

where $\lambda_b, \lambda_L$ are the inverses of the $K^0, K^0$ lifetimes and $\Delta m$ is the mass difference. This result is independent of the validity of CP invariance for the neutral $K$ decays. If all times are measured in units of the $K^0$ lifetime, we have

$$I(t)=e^{-t}+e^{-t}+2e^{(t+\alpha t)}\cos\Delta\omega t,$$

(2)

where $\alpha=\lambda_b/\lambda_L$ and $\Delta\omega=(\Delta m^2/\hbar)\lambda_b$, the mass difference in units of the inverse $K^0$ lifetime, which is the customary way for reporting the results of such an experiment.

Previous measurements$^2$-11 of $\Delta\omega$ are tabulated in Table I; the first three use this method, while the next five are based on observing the interference of $K^0$ with regenerated $K^0$, by monitoring the $\pi^+\pi^-$ decay mode. The last comes from interference of $K^0$ and $K^0$ leptonic decay.

The source of $K^0$ for our experiment was charge-exchange scattering of $K^-$ in hydrogen and deuterium.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure1.png}
\caption{Schematic representation of $K^0$, $K^0$, $K^0$ amplitudes, showing origin of Eq. (1) in the text. Convention that $\epsilon^{*}=(|K^0|'-|K^0|)/2$ is assumed. Note that intensity of $K^0$ is independent of this convention and also of the sign of $\Delta\omega$.}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure2.png}
\caption{Event topologies for this experiment. Vertices designated by letters $A, B, C$ tracks connected to one vertex by $A1, B1$ etc. connecting tracks $AB, BC$.}
\end{figure}

II. SELECTION OF EVENTS

About 200,000 pictures from the LRL 25-in. hydrogen chamber were searched for events of the topologies shown in Fig. 2. Events with these topologies could also include a number of interactions of Λ's and elastic scattering of neutral K's. About \( \frac{2}{3} \) of the approximately 300 events of these topologies can be definitely identified with a particular reaction; the rest are presumably underconstrained modes or unrelated vertices.

Figure 3 is a photograph of an actual event. Beam momenta ranged from 850 to 1150 MeV/c.

### Table I. Measurements of \( \Delta \omega \).

<table>
<thead>
<tr>
<th>Reference</th>
<th>( \Delta \omega )</th>
<th>Method</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fitch et al.</td>
<td>1.9 ±0.3</td>
<td>Strangeness versus time</td>
</tr>
<tr>
<td>Camerini et al.</td>
<td>1.5 ±0.2</td>
<td></td>
</tr>
<tr>
<td>Erratum to 3</td>
<td>0.88 ±0.2</td>
<td></td>
</tr>
<tr>
<td>Meisner et al.</td>
<td>0.63±0.20</td>
<td></td>
</tr>
<tr>
<td>This experiment</td>
<td>0.50±0.15</td>
<td></td>
</tr>
<tr>
<td>Good et al.</td>
<td>0.55±0.15</td>
<td>Coherent regeneration</td>
</tr>
<tr>
<td>Christenson et al.</td>
<td>0.52±0.15</td>
<td>Interference of ( K_{L1}^{*} ), re-</td>
</tr>
<tr>
<td>Fujii et al.</td>
<td>0.45±0.06</td>
<td>generated ( K_{L1}^{*} ) 2x modes</td>
</tr>
<tr>
<td>Vishelevsky et al.</td>
<td>0.35±0.15</td>
<td></td>
</tr>
<tr>
<td>Aiff-Steinberger et al.</td>
<td>0.78±0.20</td>
<td>Time dependence of leptonic decay</td>
</tr>
</tbody>
</table>

Weighted mean, this experiment \(\,^a\) | 0.63±0.11 |                                     |

Weighted mean \(\,^a\) | 0.51±0.04 |                                     |

### Table II. Reactions leading to events in sample.

<table>
<thead>
<tr>
<th>Hydrogen production</th>
</tr>
</thead>
<tbody>
<tr>
<td>(A) ( K^-+p \rightarrow \bar{K}^0+n )</td>
</tr>
<tr>
<td>(B) ( K^-+p \rightarrow \bar{K}^0+n^-+p )</td>
</tr>
<tr>
<td>(C) ( K^-+p \rightarrow \bar{K}^0+n^++n )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Deuterium production</th>
</tr>
</thead>
<tbody>
<tr>
<td>(D) ( K^-+p(+n) \rightarrow \bar{K}^0+n(+n) )</td>
</tr>
<tr>
<td>(E) ( K^-+p(+n) \rightarrow \bar{K}^0+n^-+p(+n) )</td>
</tr>
<tr>
<td>(F) ( K^-+n(+p) \rightarrow \bar{K}^0+n^-+n(+p) ) spectator visible</td>
</tr>
<tr>
<td>(P) Same with no visible spectator</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Hydrogen ( S=-1 ) interactions</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a) ( \bar{K}^0+n \rightarrow \Lambda + \pi^+ )</td>
</tr>
<tr>
<td>(b) ( \bar{K}^0+n \rightarrow \Lambda + \pi^- + \pi^0 )</td>
</tr>
<tr>
<td>(c) ( \bar{K}^0+n \rightarrow \Sigma^0 + \pi^- )</td>
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</tbody>
</table>

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<td>(d) ( \bar{K}^0+n(+p) \rightarrow \Lambda + \pi^- (+n) )</td>
</tr>
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<td>(e) ( \bar{K}^0+n(+p) \rightarrow \Sigma^0 + \pi^0 (+p) ) spectator visible</td>
</tr>
<tr>
<td>(P) Same with no visible spectator</td>
</tr>
</tbody>
</table>
Table II lists the possible strangeness = −1 reactions employed in this experiment. Two criteria were imposed on all events: (i) An acceptable simultaneous 2- or 3-vertex kinematic fit (computer program HASH), at the highest level in which all vertices are determined, and (ii) ionization of the recoil particle (particle B1 in Fig. 2) compatible with a π and incompatible with a proton. Because of the low momenta of B1 tracks, this notation see Fig. 2) criterion (ii) was nearly always applicable; only two events were discarded as being ambiguous. Events with predicted proton ionization between 1.4 and 2.5 times minimum were bubble counted; those with proton ionization less than 1.4 were discarded, and those over 2.5 were obvious from inspection and needed no bubble count. On deuterium events with neutron spectators at either or both of the first two vertices, an additional criterion was applied: (iii) The spectator momentum determined from the kinematic fit must be less than 225 MeV/c (95% of the events from a Hulthén distribution should be below this value).

The spectator momentum distribution of events passing criteria (i) and (ii) is plotted in Fig. 4. This criterion has the effect of imposing an additional weak kinematic constraint; it undoubtedly leads to rejection of a few good events, but helps keep the sample "clean." Table III lists the possible kinematic classifications of complete events in this experiment, and the number of events found in each class. All other conceivable classes were rejected as insufficiently constrained to permit a definitive test.

The x² cutoffs were chosen at the 98% confidence level, and then multiplied by a factor 1.4. This adjustment is based on previous experience with the analysis programs used, where we find all constraint classes tend to give mean values of x² too high by this factor. The mean values of x² in the sample were consistent with this experience. Only seven events were rejected with x² between the cutoff and twice the cutoff value. Only events in category VII had no information from the production vertex (though its position did establish the R³ direction). It was felt that in this case the constraints were sufficiently tight with two vertices alone, and the number of events found in this category was about what we would have expected. These events were required to have R³ moments within the limits imposed by production reaction (C).

All events were also fitted to appropriate competing hypotheses, involving Λ interactions. These could also be dismissed by the ionization tests, except for the reaction

\[ \Lambda + p \rightarrow \pi^+ + \Lambda + n. \]

Most of the Λ's in this film are below threshold for this reaction; only one event was found to be compatible with it, and was discarded from the sample.

The 45 events in categories with 5 or more kinematic constraints (see Table III) comprise an extremely clean sample. Only one event of this type had an acceptable competing kinematic hypothesis; this event was one of those discarded because the momentum of track B1 was too high for an unambiguous ionization measurement. Only one event with acceptable kinematics failed the ionization test.

The 32 events with 4 or less kinematic constraints are less ambiguous. Fifty-five events in these classes passed the kinematics test, but 23 were discarded for having ionization consistent with a Λ-p scattering. Eleven of the 32 accepted had acceptable fits to Λ scattering, but the ionization test clearly indicated that track B1 was a π⁺.

In order to fit events from production reaction (D) in deuterium, a Monte Carlo program was used to generate R³ by this reaction at the beam energies used in the experiment, using a Hulthén wave function for the deuteron. The resulting distribution of R³ momentum versus angle was tabulated and assigned as incoming R³ momentum at vertex B1, with error assigned from the rms spread of the values obtained from the Monte Carlo program. This spread was reasonably close to Gaussian, and for events in our sample the assigned

\[ M. Gourdin and A. Martín, Nuovo Cimento 11, 670 (1959). \]
errors in momentum ranged from 5\% to 12\%, adequate to provide a reasonably tight kinematic fit.

Deuteron events with spectator protons too low in momentum to be observed \( \langle \text{production reaction (} F' \rangle \) of Table II] were fitted by assuming the reaction occurred on a free neutron, with slightly increased errors on the input data. In these reactions the proton spectator must have momentum less than about 80 MeV/c, and most of the good events would pass such a fit.

Four events in the sample had acceptable fits in more than one class; in each case, they were assigned to the class with the most probable value of \( \chi^2 \). In no case was the difference in \( \vec{K}^0 \) momentum values sufficient to materially affect the result of the experiment. This was verified by twice re-analyzing the data using all the highest and all the lowest solutions, respectively; in each case \( \Delta \omega \) was shifted by less than 0.003.

For purposes of the analysis, all vertices were required to fall within a fiducial volume about 4 cm smaller than the visible chamber, and neutral connecting tracks had to be at least 0.2 cm in length. Six events otherwise acceptable were rejected for failing to meet these criteria.

### III. BACKGROUND AND BIASES

As indicated in the preceding section, every event in the sample has had both an acceptable kinematic fit and an unambiguous ionization test. Only 11 of the 77 events in the final sample could have another kinematic interpretation, and this has been ruled out by the ionization test. Thus the background from events of the same topology but not representing a \( \vec{K}^0 \) interaction is almost certainly negligible.

An additional possible source of background is genuine \( \vec{K}^0 \) interactions of \( K_L^0 \) originating outside the chamber. During the scan a number of events were found with "incomplete" topology (vertices B and C of Fig. 2 present, but no production vertex). These were roughly as numerous as our final sample. An exact comparison is difficult because these events were not subjected to a complete analysis. We feel that such an event will rarely give an acceptable fit to an "unassociated" production vertex. A background of this type would appear primarily at long flight times, where its presence would be easily detected, and would affect the fit for \( \Delta \omega \) very little. As an admittedly weak verification of the efficiency of kinematic fitting for rejecting this background, six events in the sample had more than one potential origin in the chamber; all six fit at one origin and failed at the other.

The only scanning bias that could affect the result of this experiment would be one dependent on the \( \vec{K}^0 \) time of flight, i.e., the separation of vertices A and B. Such a bias is quite conceivable, especially if the scanning efficiency is low. A rescans of about \( \frac{1}{3} \) of the film, containing 123 candidates with the topology used in this experiment, yielded 11 events missed on either one scan or the other; thus we estimate our scanning efficiency on a single scan as \( 1-11/2 \times 123 = 0.95 \). Of course this estimate is not valid if there exist significant numbers of events that are unusually hard to find.

A third possible source of difficulty is kinematic fitting bias. This could introduce a false time dependence if the angle error-length relations assumed for neutral tracks in the kinematic fits were incorrect. It was to reduce the possibility of such a bias that high-\( \chi^2 \) cutoffs were chosen.

### IV. DATA ANALYSIS

The events in the final sample were analyzed by the maximum-likelihood method, normalizing to "potential time" as follows: for each event we compute the function

\[
F(\Delta \omega, t) = \frac{I(\Delta \omega, t)}{I(\Delta \omega, t') dt'},
\]

where \( t_0 \) is the (proper) time at which the neutral \( K \) passed the 0.2-cm minimum-path-length requirement, and \( t_m \) is the (proper) time corresponding to the maximum-possible neutral-\( K \) path length that would leave both the secondary vertices in the fiducial volume, taking into account the position of the origin and the \( \vec{K}^0 \) direction. As the mean value of \( t_m \) for our sample is about 12 \( K_L^0 \) lifetimes, any error in estimating \( t_m \), such as assuming too large a fiducial volume, will affect the normalizing integral in a way that is not very sensitive to \( \Delta \omega \). The likelihood function is then computed from the actual proper times of flight \( t_i \) of the events

\[
L(\Delta \omega) = \prod_i F_i(\Delta \omega, t_i),
\]

the product being evaluated for various values of \( \Delta \omega \). This function is plotted in Fig. 5.

From the likelihood function, we obtain

\[
\Delta \omega = 0.50 \pm 0.15,
\]

the errors representing the usual \( e^{-1/2} \) convention. The expected time distribution of the events is obtained by summing the \( F_i \)

\[
W(\Delta \omega, t) = \sum_i F_i(\Delta \omega, t).
\]
This function then represents the theoretical time distribution of events over the observed set of potential times. This function is plotted in Fig. 6, for \( \Delta \omega = 0, 0.5 \), and 1.0, along with a histogram of the actual time distribution. Some of the detail of the curves represents the structure in the distribution of potential times of observation, since this is obtained from the actual sample of events.

The excess of events in the first three lifetimes is somewhat disturbing, but it is within statistical possibility; the distribution, compared with the “theoretical” curve for \( \Delta \omega = 0.5 \) gives \( \chi^2 = 9.2 \) for 6 degrees of freedom, which has 17% probability. (The bins for this \( \chi^2 \) were lifetimes 1, 2, 3, 4–6, 7–9, 9–12, and 12–18.) The loss of a few events near the edge of the fiducial volume might account for this effect. Since this occurs at long times, the value of \( \Delta \omega \) would not be very sensitive to this loss. If this effect represents a time-dependent bias, the true value of \( \Delta \omega \) should be somewhat higher than we obtain.

As a check both on the identification of events and time-dependent biases, the 71 events leading to \( \Lambda \) decay were used to evaluate the \( \Lambda \) mean lifetime, also using a potential-time likelihood function and the same fiducial volume. We obtain

\[
\tau_\Lambda = (2.32 \pm 0.31) \times 10^{-10} \text{ sec},
\]

which is in reasonable agreement with the current world average\(^1\) of \((2.61 \pm 0.02) \times 10^{-10} \text{ sec}. \) Thus there are


probably no severe time-dependent biases on the identification of the \( \Lambda \), which is as one would expect, in view of the high scanning efficiency. Of course it is possible to imagine time-dependent scanning biases that would affect the \( K^0 \) reaction-time distribution but not that of the \( \Lambda \) decay.

Several additional checks on the internal consistency of the data were performed. First, the likelihood program was run only on a sample of 51 kinematically clean hydrogen events (category VII excluded). This gave \( \Delta \omega = 0.40 \pm 0.22 \). To check the insensitivity of the data to the choice of fiducial volume for the potential time calculation, the data were rerun with the chamber assumed 2 cm smaller in all dimensions, but without changing the events in the sample. This caused a shift of only 0.02 in \( \Delta \omega \). As a further check, the observed distribution of potential times was compared with one predicted by a Monte Carlo calculation; the agreement was excellent.

V. DISCUSSION

It is apparent that, with the exception of Ref. 2 and taking into account the revised result of Ref. 3, all measurements of \( \Delta \omega \) by the strangeness-time method are now reasonably consistent with each other and with the values obtained by regeneration methods. Forming a combined result from Refs. 4 and 5 and this experiment, we obtain a weighted mean

\[
\Delta \omega = 0.63 \pm 0.11 \text{ (strangeness versus time)}.
\]

A similar average may be formed from the experiments in Refs. 6 through 10.

\[
\Delta \omega = 0.51 \pm 0.04 \text{ (regeneration)}.
\]

On the present level of statistical accuracy, we conclude that there is no reason to believe that the value of the mass difference depends on the method employed.

ACKNOWLEDGMENTS

The authors would like to thank the staffs of the Bevatron and the 25-in. hydrogen chamber, the many members of the Powell-Burge group at the Lawrence Radiation Laboratory that helped with the run, and our staff of scanners and measurers.
Fig. 3. Photograph of a typical event (from class II of Table III).