WEAK MAGNETISM AND THE BETA SPECTRA OF $^{12}$B AND $^{12}$N

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Abstract: We perform a careful analysis of the shapes of the beta spectra for Gamow-Teller decays and apply this to the previously measured $^{12}$B, $^{12}$N cases. The results are found not to support the CVC hypothesis as strongly as previous studies appeared to indicate.

1. Introduction

Measurements of the weak magnetism form factors in the mass-12 $\beta$-decays constitute a critical test of the conserved vector current (CVC) hypothesis $^1$. The shapes of the beta spectra for $^{12}$B and $^{12}$N and their relationship to weak magnetism have been discussed by many authors $^2$ and the results have been found to be in agreement with the predictions of CVC. Recently, however, a similar shape factor measurement was made for the decay of $^{20}$F and the results seem to be at variance with the value predicted by CVC $^3$. In addition an angular correlation measurement in the mass-12 system requires second class currents if CVC is valid $^4$. It is therefore of interest to reexamine the crucial mass-12 system.

We note that since the time when the original measurements were performed there has been a considerable change in the experimental M1 width used in the CVC prediction $^5$. Also the endpoint energies are now known with greater precision $^6$. The influence of both effects will be assessed below. We include a careful analysis of the normalization used in the experiment and of recoil effects associated with the leading Gamow-Teller term which are usually neglected but which can be of importance. Finally we perform a calculation of Coulomb effects for this system and correct some of the confusion and double counting which has sometimes occurred in the past.

The spectra of the mass-12 system also offer a testing ground for the existence of a component of the polar vector current which is odd under the G-parity operation, a second class vector current $^7$. As recently emphasized by Holstein and Treiman $^8$, the other CVC tests, pion $\beta$-decay and $0^+\rightarrow0^+$ analog decays, are inherently in-

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sensitive to the presence of such a current. The fact that these tests verify CVC indicates that the first class component of the current is conserved but says nothing about the possible existence of a second class partner. If such a current contributes to the weak magnetism term in a non-analog decay \(^{1}\) we will find

\[
\begin{align*}
    b(e^-) &= b^I + b^{II}, \\
    b(e^+) &= b^I - b^{II},
\end{align*}
\]

where \(b^I (b^{II})\) is the contribution to weak magnetism from a first class (second class) vector current. Now CVC requires \(b^I\) to be related to the analog electromagnetic M1 decay form factor \(b_\gamma\). Thus even if agreement with the strict CVC prediction is not obtained for the electron branch, it is still conceivable that a weak form of CVC holds as discussed above. If so, the positron branch will obey eq. (1). We study this possibility below.

A brief outline of this work is as follows. In sect. 2 we examine the theoretical spectra with and without Coulomb corrections. Sect. 3 summarizes the numerical aspects of the mass-12 system and presents the current status of the CVC tests. Finally in sect. 4 we suggest possibilities for further work in this line.

2. Theoretical analysis of the momentum spectrum

We begin by summarizing the form of the theoretical spectrum. In order to simplify the discussion we temporarily neglect electromagnetic effects. The momentum spectrum for unpolarized nuclei is then given by the expression

\[
\begin{align*}
    d\lambda &= \left[ \frac{4}{(2\pi)^3} G^2 \cos^2 \theta_C \right] p^2 (E_0 - E)^3 h_1(E) dp,
\end{align*}
\]

where \(G(\sim 10^{-5} m_p^2)\) is the weak coupling constant, \(\theta_C(\approx 15^\circ)\) is the Cabibbo angle, \(p(E)\) is the electron momentum (energy), and

\[
E_0 = \Delta(1 + m_e^2/2\Delta)/(1 + \Delta/2M)
\]

is the maximum value of this energy. The quantities \(\Delta\) and \(M\) are given by

\[
\Delta = M_1 - M_2, \quad M = \frac{1}{2}(M_1 + M_2)
\]

where \(M_1\) and \(M_2\) are the masses of the parent and daughter nuclei, respectively.

The spectral function \(h_1(E)\) for a Gamow-Teller decay is

\[
\begin{align*}
    h_1(E) &= \tilde{h}_1(E) + \frac{2}{3} c_1 c_2 (11m_e^2 + 20EE_0 - 20E^2 - 2m_e^2E_0/E), \\
    \tilde{h}_1(E) &= c_1^2 - \frac{2}{3} \frac{E_0}{M} c_1(c_1 + d \pm b) + \frac{2}{3} \frac{E}{M} c_1(5c_1 \pm 2b) \\
    &\quad - \frac{m_e^2}{3ME} [2c_1^2 + c_1(d \pm 2b) - c_1 h(E_0 - E)/M],
\end{align*}
\]

\(^{1}\) Note that for an analog \(\beta\)-decay \(b^{II} = 0\) by symmetry arguments while for a non-analog transition the impulse approximation requires the vanishing of \(b^{II}\). The latter condition is no longer true, however, in the presence of exchange currents.
where the upper (lower) sign is for electron (positron) decay and the terms $c, b, d,$ etc. are weak structure functions or form factors. Form factor $b$ arises from the polar vector piece of the weak current and is the weak magnetism term, while structure functions $c, d$ and $h$ arise from the axial vector current and are the Gamow-Teller, axial tensor and induced pseudoscalar form factors, respectively. Each of these is a function of momentum transfer $q^2$ but for nuclear $\beta$-decay $q$ is sufficiently small that this feature is relevant only for the leading Gamow-Teller term $c$, for which we have written

$$c(q^2) = c_1 + c_2 q^2 + \ldots .$$  (5)

The electromagnetic corrections to this result can be split into two components. First the spectrum is multiplied by the function $R(E, E_0)$, a correction factor for bremsstrahlung and for additional radiative corrections not included in the Coulomb wave function. To first order in $a$ it has the form $^{11)}$

$$R(E, E_0) = 1 + \frac{a}{2\pi} g(E, E_0),$$  (6)

where

$$g(E, E_0) = 3 \ln \left( \frac{m_p}{m_e} \right) - \frac{3}{4} + 4 \left( \frac{1}{\beta} \tanh^{-1} \beta - 1 \right) \left[ \frac{E_0 - E}{3E} - \frac{3}{2} + \ln \frac{2(E_0 - E)}{m_e} \right]$$

$$+ \frac{4}{\beta} L \left( \frac{2\beta}{1 + \beta} \right) + \frac{1}{\beta} \tanh^{-1} \beta \left[ 2(1 + \beta^2) + \frac{(E_0 - E)^2}{6E^2} - 4 \tanh^{-1} \beta \right].$$  (7)

Here $\beta = p/E$ and $L(x)$ is the Spence function

$$L(x) = \int_0^x \frac{dt}{t} \ln(1 - t).$$  (8)

Of course, this expression deletes certain energy independent electromagnetic corrections of order $a$ which merely serve to renormalize the form factors $c, b, d,$ etc. In addition, we omit higher order effects proportional to $Z a^2$, $(Z a)^2$, which modify the function $R(E, E_0)$ and the form factors $^{12)}$. Secondly, there is a correction due to the fact that the electron is in a Coulomb potential. In the appendix we demonstrate that use of a Coulomb solution to the Dirac equation for $j = \frac{1}{2}$ and $j = \frac{3}{2}$ electron waves leads to the form $^{13)}$

$$h_1(E) \to \tilde{h}_1(E) \left[ |A|^2 + |B|^2 + |C|^2 + |D|^2 + \frac{3}{4} \Re \left( A* D + B*C \right) \right.$$

$$+ \frac{2m_e}{E} \Re \left( A*B + C*D + \frac{1}{3} A*C + \frac{1}{3} B*D \right) - \frac{2}{9} \frac{p}{E} \Re \left( A*F - B*G + 3F*D - 3G*C \right) \left]. \right.$$  (9)

where $\tilde{h}_1(E)$ is given in eq. (4) and $A, B, \ldots G$ are integrals of the lepton wave functions weighted by the corresponding “weak charge” density and are defined in the appendix.
In principle the problem is completed at this stage, as the integrals can be performed numerically and the results compared with experiment. However, it is useful to have approximate analytic forms for these Coulomb corrections for use in comparing our calculations with previous work in this area. Generally the results are written in the form

\[ d\lambda = \frac{4}{(2\pi)^3} G^2 \cos^2 \theta_c (E_0 - E)^2 p^2 R(E, E_0)F(Z, E)[h_1(E) + \delta h_1(E)] dp, \]  

(10)

where \( F(Z, E) \) is a Fermi function and \( \delta h_1(E) \) are additional Coulomb corrections not included in the Fermi factor. A problem is that there exist at least three separate definitions of the Fermi function. The oldest is the point charge function \( F_0(Z, E) \)

\[ F_0(Z, E) = 4(2pR)^{2(\gamma_1 - 1)}e^{\gamma_1 y} \frac{|F(\gamma_1 + iy)|^2}{(F(1 + 2\gamma_1))^2}; \]

\[ \gamma_1 = \sqrt{1-(\alpha Z)^2}, \quad y = \pm \alpha ZE/p. \]  

(11)

Here \( Z \) is the charge of the daughter nucleus and is by definition positive. More recently an alternative definition for \( F(Z, E) \) has been used. A distributed charge Fermi function has been defined in terms of the Coulomb solutions \( g_{-1}(r) \) and \( f_1(r) \) for the electron wave function

\[ F(Z, E) = (f_1^2 + g_{-1}^2)/2p^2. \]  

(12)

The functions \( g_{-1}, f_1 \) are calculated numerically and are evaluated either at the nuclear radius or at the nuclear center. The former is done in the Bhalla-Rose table \(^{14}\) while the latter is used by Behrens and Jänecke \(^{15}\). In either case, an appropriate modification of the spectral function \( h_1(E) \) is still needed to take account of the fact that the electron wave function (for a distributed charge) must be averaged over the nuclear volume and to account for corrections due to the "small" Coulomb solutions \( f_{-1}, g_1, \) etc.

From the results of the appendix we find that if the Behrens-Jänecke Fermi function is used, the appropriate modification of the spectral function \( h_1(E) \) is given by

\[ \delta h_1^0(E) \approx \left( \pm \frac{4}{3} \frac{\alpha Z E_0}{R} - \frac{9}{2} \left( \frac{\alpha Z}{R} \right)^2 \mp \frac{22}{3} \frac{\alpha Z E}{R} \right) c_1 c_2 + \ldots \]  

(13)

On the other hand if the point charge Fermi function is utilized, the spectral modification becomes

\[ \delta h_1^0(E) \approx \pm \frac{13}{15}\alpha Z E R c_1^2 + \delta h_1^0(E) + \ldots, \]  

(14)

where we have the result that to this order \(^{16}\)

\[ F_{\text{Behrens-Jänecke}}(Z, E) = F_0(Z, E)(1 \mp \frac{13}{15}\alpha Z E R + \ldots). \]  

(15)
Finally, if the Bhalla-Rose Fermi function is employed, the spectral modification can become very large and includes a sizeable non-Coulomb component

$$\delta h'_1(E) = \left[ \pm \frac{1}{15} 6 \alpha Z R + \frac{1}{3} (E^2 - m_e^2) R^2 + \frac{1}{30} (\alpha Z)^2 \right] c^2 + \delta h'_1(E)$$

where we have used the result that

$$\delta h'_1(E) = \left[ \pm \frac{1}{15} 6 \alpha Z R + \frac{1}{3} (E^2 - m_e^2) R^2 + \frac{1}{30} (\alpha Z)^2 \right] c^2 + \delta h'_1(E).$$

It is useful to plot the various Fermi function definitions in order to visualize the difference between them. This we have done in figure 1 where we plot

![Figure 1](https://example.com/figure1.png)

**Fig. 1.** A comparison of various Fermi functions $F(Z, E)$ used in the analysis of spectral data. To illustrate the differences in their dependence on the $\beta$-momentum, the point charge function ($F_0$), the Dzhelepov-Zyrianova function ($F_{DZ}$), and the Bhalla-Rose function ($F_{BR}$) are divided by the Behrens-Jänecke function ($F_{BJ}$).

$F_0(Z, E)/F_{Behrens-Jänecke}(Z, E)$ for electron and positron branches in the energy region around 10 MeV. We note that this ratio is quite linear in energy with a slight slope of $\approx \pm 0.06 \%$ MeV$^{-1}$ which agrees reasonably well with the analytical result for the slope, $\pm \frac{13}{13} Z \alpha R$, indicated in eq. (15). Quite different results are found, however, in the plot of the ratio $F_{Bhalla-Rose}/F_{Behrens-Jänecke}$. For the electron branch the mean slope is about $-0.18 \%$ MeV$^{-1}$, which agrees reasonably well with the
analytical result for the slope, \(-\frac{2}{3}ER^2 - \frac{12}{15}ZzR\), given in eq. (17). However, for positrons the slope at 10 MeV is found to be \(+0.28\%\ MeV^{-1}\) which is at considerable variance with the analytical prediction, \(-\frac{2}{3}ER^2 + \frac{12}{15}ZzR\). This is, however, just the effect of a numerical error in Bhalla and Rose's positron table which has been pointed out by Huffaker and Laird and by Bühring\(^{18}\). Note that in both cases a dominant effect on the slope is due to the non-Coulomb finite size effect. Inclusion in the spectra thus double counts finite size effects already included in the shape factor eq. (4) unless there is a sizeable correction \(\delta h_T(E)\) included as indicated in eq. (16). Thus we feel that use of the Bhalla-Rose function is inappropriate for precise spectral analysis.

No matter which form is used, it should be kept in mind that these are merely approximate analytic forms which results from an artificial splitting of the Coulomb corrections into a Fermi and spectral correction. The complete and proper expression is quoted in eq. (9).

One feature remains to be pointed out. We have in the appendix calculated the leading (Gamow-Teller) Coulomb effects and assumed them to be identical for the recoil parts of the spectrum. This approximation is quite good in general. Corrections are (i) of order \(ZaqR(q/m)\) and hence negligible and (ii) of order \(Zz(1/mR)\). The latter are termed "induced" Coulomb corrections and their importance has been emphasized by Bottino, Ciocchetti and Kim\(^{19}\). For our case the induced Coulomb modification can be written as a correction to the spectral function \(h_1\)

\[
\delta h_1(E) \approx \frac{\sqrt{10}}{6} \frac{\alpha Z}{MR} c_1(2b + d^1 \pm d^2 \pm c_1),
\]

where \(d^1 (d^{11})\) is the axial-tensor form factor arising from first (second) class currents.

Note we have omitted completely two additional effects, the electron screening correction and the effects of the diffuse nuclear surface. The former effect, however, is seen to be small for mass-12 in the tables of Behrens and Jänecke\(^{15}\), while the latter was shown to be quite minor in the work of Asai and Ogata\(^{20}\).

### 3. Experimental data and the nuclear form factors

In the following we make use of experimental data to determine \(c_1\) and \(b_\gamma\) in order to calculate the spectral function \(h_1(E)\). In doing so, we follow tradition in that the \(b\)-term is determined from the radiative width of the analog \(\gamma\)-decay, by virtue of CVC, and we then compare the predicted energy dependence of \(h_1(E)\) with the experimentally determined value. We also present the more direct, and equivalent, comparison of experimental values of \(b(e^-)\), \(b(e^+)\) and \(b_\gamma\). As discussed earlier, these should be equal if the strong statement of CVC is correct.

**The M1 radiative width.** The \(1^+ \to 0^+ \gamma\)-decay from the \(T = 1, 15.106\) MeV level of \(^{12}\)C to the ground state is the analog of the main \(\beta\)-transitions of \(^{12}\)B and \(^{12}\)N.
The transition is a pure M1 multipole which is essentially isovector and the radiative width is related to the magnetic form factor, at zero momentum transfer, by the expression

$$\Gamma_{M1} = \frac{1}{2} \alpha E_\gamma^2 \beta^2 / M^2.$$  (19)

Here $\alpha$ is the fine structure constant, $E_\gamma$ is the photon energy and $M$ is the nuclear mass.

Measurements of the radiative width have been made by inelastic electron and photon scattering. A summary discussion of these data is given by Chertok et al. who show that the results are now in good agreement. We use the electron scattering value measured by Chertok et al., $\Gamma_{M1} = 37.0 \pm 1.0$ eV, which is somewhat smaller than the value used by Wu in an earlier summary of the status of tests of the CVC theory, $\Gamma_{M1} = 53 \pm 1.1$ eV. With $\Gamma_{M1} = 37.0 \pm 1.0$ eV, $E_\gamma = 15.110 \pm 0.003$ MeV [ref. 22] and $M = 11178$ MeV we obtain $b_\gamma(0) = 33.2 \pm 0.4$.

We have so far neglected the effect of a possible isospin impurity in the analog $T = 1$ state of $^{12}$C. A $T = 0$ impurity in this state with an amplitude of 0.1 has been suggested to explain the $^{12}$C(d, d')$^{12}$C* isospin-forbidden reaction to this level. Such an impurity implies that the radiative width has also an isoscalar as well as the desired isovector component. If we assume that the ratio of isovector to isoscalar form factors is the same as the ratio of corresponding nucleon spin $g$-factors, we obtain $b_{\gamma}^{T=1} = 1.03 b_\gamma$. This is a change which is slightly larger than the experimental error in $b$. Since the correction to $b$ is model dependent, we regard it as having an uncertainty of 100% and combine this uncertainty in quadrature with the 0.4 error in $b$ to obtain finally

$$b_{\gamma}^{T=1} = 34.2 \pm 1.1$$  (20)

The $Ft$ values for $^{12}$B and $^{12}$N and the form factor $c_1$. We next make use of the $Ft$ values calculated by Wilkinson to determine $c_1$. The non-Coulomb $c_2$ term and the $F_{Bj}(Z, E)$ and $R(E, E_0)$ are absorbed into Wilkinson’s definition of the $Ft$ value. Thus, the $Ft$ value is related to the energy average of $h_1(E)$, obtained by combining eqs. (4.13) and (4.18), by the following

$$(Ft)_{e^+} = \frac{2Ft(Fermi)}{h_1(E)_{e^+}},$$

where

$$h_1(E) = c_1^2 \left[ 1 - c_2 \left( \frac{9}{2} \left( \frac{\alpha Z}{R} \right)^2 + \frac{7}{3} \frac{\alpha Z E_0}{R} \right) \right] + \frac{\sqrt{10}}{6} \frac{\alpha Z}{M R} \left( 2 \frac{b}{c_1} + \frac{d^{ui}}{c_1} \pm \frac{d^i}{c_1} \pm 1 \right) - \frac{2}{3} \frac{E_0}{M} \left( 1 + \frac{d^i}{c_1} \pm \frac{d^{ii}}{c_1} \right).$$  (21)

In table 1 we summarize the experimental $Ft$ values for $^{12}$B and $^{12}$N, given in ref. 24), and the values of $h_1(E)$ determined from the $Ft$ values as specified above with $Ft$
The spectrum shape factors. We now examine measurements of the momentum spectra for the purpose of determining the weak magnetism form factors $b(e^-)$ and $b(e^+)$. In doing this we first correct the published data for two effects: (i) the change in the shape factor when the Behrens-Jänecke Fermi function is used and (ii) the influence of the improved endpoint energy on the phase space factor and its effect on the determination of the spectral function $h_1(E)$.

For the Fermi function correction we emphasize again that there are relatively large Coulomb corrections in the Fermi function and in the spectral shape function $h_1(E)$ and these must be handled in a consistent way (see sect. 2). To simplify matters we choose to quote the experimental shape factors with respect to a common Fermi function, the Behrens-Jänecke Fermi function, $F_B$. In table 2 we present a summary of the Fermi functions used by the three experimental groups who have measured the mass-12 spectra. We also give corrections which must be added to the slopes of their

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**Table**
The $F_t$ values and the form

<table>
<thead>
<tr>
<th>Decay</th>
<th>$E_0$ (keV)</th>
<th>$t_{1/2}$ (msec $^a$)</th>
<th>$F_t$ (sec $^a$)</th>
<th>$f_t(E)$$_{exp}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{12}$B(e$^-$)</td>
<td>13872.5 $\pm$ 1.3</td>
<td>21.01 $\pm$ 0.09</td>
<td>11789 $\pm$ 51</td>
<td>0.5229 $\pm$ 0.0023</td>
</tr>
<tr>
<td>$^{12}$N(e$^+$)</td>
<td>16818.5 $\pm$ 5.0</td>
<td>11.61 $\pm$ 0.07</td>
<td>13151 $\pm$ 82</td>
<td>0.4687 $\pm$ 0.0030</td>
</tr>
</tbody>
</table>

$^a$) Ref. 24.

(Fermi) = 3087.9 $\pm$ 3.5 sec [ref. 26]). We also give the numerical relationship between $h_1(E)$ and $c_1^2$ by evaluating the higher order terms in eq. (21). For this purpose, we use $c_2/c_1 = 1.55 F^2$ obtained from an analysis of inelastic electron scattering $^{28}$ (see sect. 4, however), $R = 1.2 A^{4/3} F$, $d^{ll}/c_1 = +44$ (shell-model calculation) $^{27}$ and two values of $d^{ll}/c_1$, 0 and $-40$. The two values of $d^{ll}/c_1$ are chosen to illustrate the sensitivity of this analysis to the possible existence of second class currents. No second class axial current implies $d^{ll} = 0$ whereas the recent measurement of the $\beta$-spin correlations on this system by Sugimoto et al. $^4$, which determines $(d^{ll}/c) - (b/c)$, implies $d^{ll}/c = -40 \pm 13$ provided $b = b_r$. The latter value of $d^{ll}$ could account for a large part of the $F_t$ asymmetry but a charge dependent and second class part of $c_1$ could also contribute to the asymmetry. We express the final result for $c$ in terms of $b$ since the latter form factor is to be determined. Note, however, that the dependence of $c$ on the $b$-term is only 0.6 $\%$, if we use $b_r$, and on $d^{ll}$ it is $\approx$ 1 $\%$. We could regard these effects as uncertainties in the experimental determination of $c_1$. 

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factor $c_1$ for $^{12}$B and $^{12}$N

<table>
<thead>
<tr>
<th>$f_i(E)/c_1^2$</th>
<th>$c_i/(1 - 0.0015 b/c_1)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$d''/c_1 = -40$</td>
<td>$d''/c_1 = 0$</td>
</tr>
<tr>
<td>0.9901 + 0.0003 $b/c_1$</td>
<td>0.9603 + 0.0003 $b/c_1$</td>
</tr>
<tr>
<td>0.9138 + 0.0003 $b/c_1$</td>
<td>0.9556 + 0.0003 $b/c_1$</td>
</tr>
</tbody>
</table>

published shape factors to convert them for use with the Behrens-Jänecke table. Mayer-Kukuk and Michel $^{28}$) used the point charge Fermi function. Glass and Peterson $^{29}$) measured the ratio of the shapes of the spectra and in their analysis they omit the Fermi function, arguing that the error is negligible. With respect to the BJ Fermi function, however, the effect is not small enough to be neglected.

The data of Lee, Mo and Wu $^{30}$) were first published using the Fermi function of Dzhelepov and Zyrianova $^{31}$). This Fermi function includes finite nuclear size effects but is tabulated only to 10 MeV. In the second publication Wu $^{21}$) used the more extensive table of Bhalla and Rose $^{14}$) which is tabulated to 20 MeV but which, unfortunately, is in error for positrons $^{17}$). Note that there is a considerable correction to the 1964 values for both positron and electron shapes. The corrections are even larger if the data are corrected to obtain shape factors defined with respect to the point charge Fermi function. We note that the point charge Fermi function should be used if the experimental results are to be compared with the spectral function of Huffaker and Laird $^{16}$) or that of Armstrong and Kim $^{13}$). Thus, in addition

### Table 2

<table>
<thead>
<tr>
<th>Exp.</th>
<th>Table</th>
<th>Slope of $F/F_{BJ}$ (‰ MeV$^{-1}$) $^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$^{12}$B(e$^-$)</td>
</tr>
<tr>
<td>Mayer-Kukuk-Michel $^b)$</td>
<td>NBS point charge</td>
<td>+ 0.064</td>
</tr>
<tr>
<td>Glass-Peterson $^c)$</td>
<td>$F = 1$</td>
<td>+ 0.085</td>
</tr>
<tr>
<td>Lee-Mo-Wu (1963) $^d)$</td>
<td>Dzhelepov-Zyrianova</td>
<td>- 0.126</td>
</tr>
<tr>
<td>(1964) $^e)$</td>
<td>Bhalla-Rose</td>
<td>- 0.171</td>
</tr>
</tbody>
</table>

$^*$ Evaluated at 10 MeV. The tabulated values of $F/F_{BJ}$ must be added to the published slopes of the shape factors to obtain a shape factor defined relative to the Behrens-Jänecke Fermi function ($F_{BJ}$).

to having the error in the Bhalla-Rose table, the comparison between experiment and theory presented by several authors also suffers from double counting some of the Coulomb finite nuclear size effects.

The second correction we make to update the published spectral shapes is associated with a change in the accepted value of the end-point energy. The correction is needed because in obtaining \( h_1(E) \) the observed counts are divided by the phase space factor, \( p^2(E_0 - E)^2 \), and an error in \( E_0 \) produces an error in the deduced spectral function \( h_1(E) \). In tables 3 and 4 we present a summary of the end-points which were used in the experiments and the corrections to the spectrum shapes needed to bring the results into line with the currently accepted value of the

### TABLE 3
End-point kinetic energy of \(^{12}\)N

<table>
<thead>
<tr>
<th>End-point (keV)</th>
<th>Measured - accepted</th>
<th>Correction to shape</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(keV)</td>
<td>((^{\circ}))</td>
</tr>
<tr>
<td>Michigan State value *)</td>
<td>16304 ± 2</td>
<td></td>
</tr>
<tr>
<td>Glass-Peterson</td>
<td>16384 ± 15</td>
<td>80 ± 15</td>
</tr>
<tr>
<td>Mayer-Kukuk-Michel</td>
<td>16360 ± 210</td>
<td>56 ± 210</td>
</tr>
<tr>
<td>(used)</td>
<td>16430 ± 60</td>
<td>126 ± 60</td>
</tr>
<tr>
<td>Lee-Mo-Wu</td>
<td>16430 ± 100</td>
<td>126 ± 100</td>
</tr>
<tr>
<td>(measured)</td>
<td>16430 ± 60</td>
<td>126 ± 60</td>
</tr>
<tr>
<td>(used 1963)</td>
<td>16320 ± 25</td>
<td>17 ± 25</td>
</tr>
</tbody>
</table>

*) Ref. 32).

### TABLE 4
End-point kinetic energy of \(^{12}\)B

<table>
<thead>
<tr>
<th>End-point (keV)</th>
<th>Measured - accepted</th>
<th>Correction to shape</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(keV)</td>
<td>((^{\circ}))</td>
</tr>
<tr>
<td>Accepted value *)</td>
<td>13361.5 ± 1.3</td>
<td></td>
</tr>
<tr>
<td>(1974 mass table)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Glass-Peterson</td>
<td>13369 ± 1</td>
<td>7.5 ± 1</td>
</tr>
<tr>
<td>(used)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mayer-Kukuk-Michel</td>
<td>13381 ± 41</td>
<td>20 ± 41</td>
</tr>
<tr>
<td>(measured)</td>
<td>13369 ± 1</td>
<td>7.5 ± 1</td>
</tr>
<tr>
<td>(used)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lee-Mo-Wu</td>
<td>13373 ± 40</td>
<td>12 ± 40</td>
</tr>
<tr>
<td>(measured)</td>
<td>13369 ± 1</td>
<td>7.5 ± 1</td>
</tr>
</tbody>
</table>

*) Ref. 33).
end-point. The main change is for $^{12}$N for which there has been a significant shift in the atomic mass excess. There has been essentially no change in the mass of $^{12}$B. The improved end-point for $^{12}$B results from using eq. (3) which takes account of the nuclear recoil in relating the end-point to the energy release.

For $^{12}$N, the differences between the currently accepted end-point and the value used by Mayer-Kukuk and Michel, and the value used in the first publication of Lee, Mo and Wu is 126 keV. In fig. 2 we illustrate the effect that such an error would have on the shape factor for a Monte Carlo sample of $^{12}$N “data”. The spectrum was generated with the complete theoretical spectrum, including the bremsstrahlung factor $R(E, E_0)$ which changes rapidly near the end-point. A weak magnetism term was put in $h_1(E)$ to produce a slope of $-0.52 \%$ MeV$^{-1}$. The open circles illustrate the shape factor when the data are analyzed with the correct mass. The solid points show what happens if the end-point is taken to be 126 keV larger than the correct value. The slope with the correct end-point is $-0.50 \pm 0.08 \%$ MeV$^{-1}$, in agreement with the assumed value, but the slope with the wrong end-point is $-1.02 \pm 0.08 \%$ MeV$^{-1}$, a difference of 0.52 \% MeV$^{-1}$.

Fig. 2. An illustration of how the observed shape factor is affected by an error in the end-point energy. The spectrum is for $^{12}$N decay and was generated by a Monte Carlo method assuming $dS/dE = -0.52 \%$ MeV$^{-1}$. The open circles illustrate the shape factor data when the correct end-point is used. The solid circles are obtained when an error of 126 keV is made in specifying the end-point energy.
It is interesting to observe that in the momentum region which was carefully investigated in the experiments the shape factor appears to be quite linear despite the use of the wrong end-point. That is, the reduced $\chi^2$ for a linear fit in this region is 0.84 with the correct end-point and still only 1.1 with the wrong end-point. As can be seen in fig. 2, however, a careful study of the spectra in the region near the end-point would have revealed that the mass of $^{12}$N was in error.

For the experiment of Lee, Mo and Wu, we give in table 3 the correction to the $^{12}$N shape factor for both of their publications. In the first publication, the shape factor was derived using the end-point which differs from the current value by 126 keV. For the second publication, an improved end-point was used which differs by only 17 keV from the current value. Using the improved end-point would have changed the slope of the shape factor by 0.43 $\%$ MeV$^{-1}$. As discussed above, however, the Fermi function was also changed between the first and second publications and this is also a significant effect for $^{12}$N, owing to the error in the Bhalla-Rose table. Since the effects of the end-point and Fermi function on the slope are comparable in magnitude, but opposite in sign, there is little difference between the two published results: $-0.62 \%$ MeV$^{-1}$ in 1963 and $-0.52 \%$ MeV$^{-1}$ in 1964.

One other feature of the published data which deserves comment is the fact that the spectral shapes for the transitions to the ground state of $^{12}$C are obtained after stripping away the weak branches to the 4.44 and 7.66 MeV levels. The spectral shapes are based on a momentum region which generally avoids decays to the 7.66 MeV level, but transitions to the 4.44 MeV level are within this momentum range. It is therefore interesting to compare the branching ratios to the 4.44 MeV level which were used in the analysis of the published data with the currently accepted values. In table 5 we summarize the branching ratio data. We observe that there has been no significant change in the branching ratios to the 4.44 MeV level and thus no need to correct the spectral shapes.

| Table 5 |
|-------------------|-------------------|
| Branching ratios in $^{12}$B-$^{12}$N decays to 4.44 MeV level of $^{12}$C |

<table>
<thead>
<tr>
<th>Experiment (year)</th>
<th>Branching ratio ($%$)</th>
<th>$^{12}$B $\rightarrow$ $^{12}$C(4.44)ev</th>
<th>$^{12}$N $\rightarrow$ $^{12}$C(4.44)ev</th>
</tr>
</thead>
<tbody>
<tr>
<td>McDonald et al. (1974)</td>
<td>1.29 ± 0.05</td>
<td>2.10 ± 0.16</td>
<td></td>
</tr>
<tr>
<td>Mayer-Kukuk-Michel (1962)</td>
<td>1.3 ± 0.1</td>
<td>2.4 ± 0.2</td>
<td></td>
</tr>
<tr>
<td>Glass-Peterson (1963)</td>
<td>1.3 ± 0.1</td>
<td>2.4 ± 0.2</td>
<td></td>
</tr>
<tr>
<td>Lee-Mo-Wu (1963, 1964)</td>
<td>1.3 ± 0.1</td>
<td>2.4 ± 0.2</td>
<td></td>
</tr>
</tbody>
</table>

*) Ref. 24).

In table 6 we present a final summary of the experimental values of the slopes of the shape factors. The original published data are given together with the values obtained after correcting for the end-point energy and Fermi function. As expected, the
WEAK MAGNETISM

Table 6
Revised slopes of shape factors of $^{12}$B-$^{12}$N decays

<table>
<thead>
<tr>
<th>Experiment</th>
<th>Decay</th>
<th>Published slope</th>
<th>Revised slope</th>
<th>Theory *)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mayer-Kukuk-Michel</td>
<td>$^{12}$B(e$^-$)</td>
<td>$+1.82 \pm 0.09$</td>
<td>$+1.92 \pm 0.09$</td>
<td>$+0.42$</td>
</tr>
<tr>
<td></td>
<td>$^{12}$N(e$^+$)</td>
<td>$+0.60 \pm 0.08$</td>
<td>$+1.07 \pm 0.08$</td>
<td>$-0.52$</td>
</tr>
<tr>
<td>Glass-Peterson</td>
<td>$^{12}$B/$^{12}$N</td>
<td>$+1.62 \pm 0.28$</td>
<td>$+1.44 \pm 0.28$</td>
<td>$+0.94$</td>
</tr>
<tr>
<td>Lee-Mo-Wu (1964)</td>
<td>$^{12}$B(e$^-$)</td>
<td>$+0.55 \pm 0.10$</td>
<td>$+0.42 \pm 0.10$</td>
<td>$+0.42$</td>
</tr>
<tr>
<td></td>
<td>$^{12}$N(e$^+$)</td>
<td>$-0.52 \pm 0.06$</td>
<td>$-0.17 \pm 0.06$</td>
<td>$-0.52$</td>
</tr>
</tbody>
</table>

*) Based on $b_s = 33.2 \pm 0.4$, $d^{II} = 0$, $c \approx 0.72$ (see tables 1 and 8). A more meaningful comparison between experiment and theory is given in table 9.

changes are rather large. The $^{12}$B-$^{12}$N shape factors of Mayer-Kukuk and Michel still have positive slopes with the $^{12}$N even more positive than before. The ratio slope of the $^{12}$B to $^{12}$N shape factors, measured by Glass and Peterson, is now smaller and the shape factors of Lee, Mo and Wu have smaller slopes which are no longer equal and opposite in sign.

Having obtained the "best values" for the shape factors, we proceed to interpret them vis à vis the weak magnetism form factor. We begin by recalling that the spectral function $h_1(E)$ which should be used with the Behrens-Jänecke Fermi function is obtained by combining eqs. (4), (13) and (18). This gives

$$h_1(E) = c_1^2 \left\{ 1 + \frac{2}{9} c_1^2 (11m_e^2 + 20EE_0 - 20E^2 - 2m_e^2E_0/E)$$

$$- \frac{c_2}{c_1} \left( \frac{9}{2} \frac{\alpha Z}{R} \right)^2 + \frac{4}{3} \frac{\alpha Z E_0}{R} \pm \frac{22}{3} \frac{\alpha Z E}{R} \right) + \frac{\sqrt{10}}{6} \frac{\alpha Z}{MR} \left( 2 \frac{b}{c_1} \pm \frac{d^I}{c_1} + \frac{d^{II}}{c_1} \pm 1 \right)$$

$$- \frac{2}{3} \frac{E_0}{M} \left( 1 + \frac{d^I}{c_1} \pm \frac{d^{II}}{c_1} \pm \frac{b}{c_1} \right) + \frac{2}{3} \frac{E}{M} \left( 5 \pm \frac{2b}{c_1} \right)$$

$$- \frac{m_e^2}{3ME} \left\{ 2 + \frac{d^I}{c_1} \pm \frac{d^{II}}{c_1} \pm \frac{2b}{c_1} - h \frac{E_0 - E}{M} \right\}. \quad (22)$$

The experimental measurements of the slope of the shape factor are obtained as follows. For each momentum, the number of observed counts is divided by the quantity $R(E, E_0)F(Z, E)p^2(E_0 - E)^2$ to obtain an unnormalized shape factor. These are then normalized to unity at some energy $E_n$ by dividing the unnormalized values by the value at $E_n$. The resulting normalized shape factor $S(E)$ is therefore given in terms of the spectral function $h_1(E)$ by

$$S(E) = h_1(E)/h_1(E_n). \quad (23)$$

The slope $dS/dE$ is the published result. Note that because of the sign change associated with the $b$, $d$ and $\alpha Z$ terms, the magnitude of the normalization factor $h_1(E_n)$ is different for electron and positron decays.
In the following analysis we neglect the terms proportional to \( m^2/ME \), since this is very small for the decays considered here, and write the function \( h_1(E) \) as follows

\[
h_1(E) = c_1^2(a_0 + a_1E + a_2E^2).
\]  
(24)

The slope \( dS/dE \) is therefore given by

\[
dS/dE = (a_1 + 2a_2E)/(a_0 + a_1E + a_2E^2).
\]  
(25)

By comparison with eq. (23) the quantities \( a_0, a_1 \) and \( a_2 \) are given by

\[
a_0 = 1 + \frac{22}{9} \frac{c_1^2}{c_2} \frac{m_e^2}{M} - \frac{c_2}{c_1} \left( 9 \left( \frac{aZ}{R} \right)^2 \pm 4 \frac{aZ}{3} E_0 \right)
\]

\[
- \frac{2}{3} \frac{E_0}{M} \left( 1 + \frac{d^i}{c_1} \pm \frac{d^m}{d_1} \pm \frac{b}{c_1} \right) + \frac{\sqrt{10}}{6} \frac{aZ}{MR} \left( 2 \frac{b}{c_1} + \frac{d^m}{c_1} \pm \frac{d^i}{c_1} \pm 1 \right),
\]  
(26)

\[
a_1 = \frac{40}{9} \frac{c_2}{c_1} \frac{E_0}{c_1} + \frac{22}{3} \frac{c_2}{c_1} \left( \frac{aZ}{R} \right) \pm \frac{4}{3} \frac{b}{Mc_1} + \frac{10}{3M}.
\]  
(27)

\[
a_2 = -\frac{40}{9} c_2/c_1.
\]  
(28)

In table 7 we give the numerical values of \( a_0, a_1 \) and \( a_2 \) based on the values of \( c_2/c_1, d^i/c_1 \) and \( d^m/c_1 \) which are given in part 2b.

**Table 7**

<table>
<thead>
<tr>
<th>Decay</th>
<th>( d^{ii}/c_1 = -40 )</th>
<th>( d^{ii}/c_1 = 0 )</th>
<th>( a_1 ) (MeV(^{-1}))</th>
<th>( a_2 ) (MeV(^{-2}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>(^{12})B(e(^-))</td>
<td>0.9970 - 0.00060 ( b/c_1 )</td>
<td>0.9671 - 0.00060 ( b/c_1 )</td>
<td>0.00204 + 0.000119 ( b/c_1 )</td>
<td>-0.000178</td>
</tr>
<tr>
<td>(^{12})N(e(^+))</td>
<td>0.9069 + 0.00120 ( b/c_1 )</td>
<td>0.9488 + 0.00120 ( b/c_1 )</td>
<td>0.00388 - 0.000119 ( b/c_1 )</td>
<td>-0.000178</td>
</tr>
</tbody>
</table>

Using eq. (25) and the values of \( a_0, a_1 \) and \( a_2 \) given in table 7 we evaluate the slope \( dS/dE \) in terms of the parameter \( b/c_1 \). The results are summarized in table 8 for the two values of \( d^{ii}/c_1 \). The slopes are evaluated at \( E = 10 \) MeV, essentially the center of the spectral data, and we have retained the small quadratic \( b \)-dependence which is introduced by the normalization.

Finally, in table 9 we present the experimental form factors. The first column gives the value of \( b/c_1 \) obtained from the revised slopes of table 6 and the theoretical functions of table 8, while the second column gives the values of the weak magnetism \( b \)-term obtained by multiplying the values of \( b/c_1 \) of col. 1 with the values of \( c_1 \) from table 1.

The individual values of \( b \) can be obtained from the data of Mayer-Kukuk and...
Table 8
Theoretical slope of the shape factors at 10 MeV

<table>
<thead>
<tr>
<th>Decay</th>
<th>( \frac{d^1}{c} = -40 )</th>
<th>( \frac{d^1}{c} = 0 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(^{12}\text{B}(e^-))</td>
<td>(-0.00176 + 0.000119(1 - 0.00008 b/c_1) b/c_1)</td>
<td>(-0.00156 + 0.000122(1 - 0.00011 b/c_1) b/c_1)</td>
</tr>
<tr>
<td>(^{12}\text{N}(e^+))</td>
<td>(+0.00004 - 0.000129(1 + 0.00050 b/c_1) b/c_1)</td>
<td>(+0.00034 - 0.000125(1 - 0.00138 b/c_1) b/c_1)</td>
</tr>
</tbody>
</table>

Table 9
Weak magnetism form factors deduced from spectrum shape *

<table>
<thead>
<tr>
<th>Exp.</th>
<th>Decay</th>
<th>( (b/c_1)_{\text{exp}} )</th>
<th>( b_{\text{exp}} )</th>
<th>( \frac{1}{2}[b(e^-) + b(e^+)]_{\text{exp}} )</th>
<th>( \frac{1}{2}[b(e^-) - b(e^+)]_{\text{exp}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mayer-Kukuk-</td>
<td>(^{12}\text{B}(e^-))</td>
<td>179</td>
<td>(130 \pm 6)</td>
<td>(+37 \pm 4)</td>
<td>(+94 \pm 4)</td>
</tr>
<tr>
<td>Michel</td>
<td>(^{12}\text{N}(e^+))</td>
<td>(-79)</td>
<td>(-57 \pm 4)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Glass-Peterson</td>
<td>(^{12}\text{B}/^{12}\text{N})</td>
<td>(65 \pm 13)</td>
<td></td>
<td>(+47 \pm 9)</td>
<td></td>
</tr>
<tr>
<td>Lee-Mo-Wu</td>
<td>(^{12}\text{B}(e^-))</td>
<td>(50 \pm 12)</td>
<td>(47 \pm 11)</td>
<td>(36 \pm 9)</td>
<td>(+23 \pm 5)</td>
</tr>
<tr>
<td>(^{12}\text{N}(e^+))</td>
<td>(13 \pm 5)</td>
<td>(17 \pm 6)</td>
<td>(9.3 \pm 3.6)</td>
<td>(12 \pm 4)</td>
<td></td>
</tr>
</tbody>
</table>

*) Compare \( b_\gamma = 33.2 \pm 0.4 \) to \( b_{\text{exp}} \) and to \( \frac{1}{2}[b(e^-) + b(e^+)]_{\text{exp}} \) for strong and weak tests of CVC, respectively.

Michel and the data of Lee, Mo and Wu. These should be compared to \( b_\gamma = 33.2 \pm 0.4 \) as a test of the strong statement of CVC. The first experiment compares very poorly with the CVC prediction. The experiment of Lee, Mo and Wu compares favorably for the \(^{12}\text{B}\) decay, but very unfavorably for the \(^{12}\text{N}\) case.

The difference of the slopes of the \(^{12}\text{B}/^{12}\text{N}\) shape factors avoids certain systematic experimental errors – an error in the calibration of the spectrometer, for example – and determines the quantity \( \frac{1}{2}[b(e^-) + b(e^+)] \). These data are presented in the third major column of table 9 and we observe that the agreement with \( b_\gamma \) is now considerably better, though the value of Lee, Mo and Wu is somewhat low.

In the last major column of table 9 we give the difference of the \( b \)-terms. This result is beset with the possibility of significant experimental errors and we should not take these data as seriously as those for the sum of the form factors. This quantity is nonetheless interesting since it is a measure of a possible second class component to the vector current, a component which, as emphasized earlier, cannot be ruled out except by these data. On the face of it, there appears to be a difference in the form factors and thus evidence for a second class \( b^{\Pi} \). However, the two experimental results of Mayer-Kukuk and Michel are inconsistent with those of Lee, Mo and Wu and a more likely explanation is therefore some systematic experimental error. We conclude that a significant second class component to \( b \) cannot be ruled out by these data.
The primary and surprising result of this analysis is that there appears to be a real problem in the comparison of the experiment of Lee, Mo and Wu with the predictions of CVC. The electron branch is in good agreement but the positron slope is considerably too small. Combining this result with the work of Calaprice and Alburger, the present evidence for the validity of CVC from shape factor measurements is rather tenuous.

4. Other possible measurements of weak magnetism

In view of the unsatisfactory status of the mass-12 spectra, it is important to consider how to improve the experimental basis of the weak magnetism feature of CVC theory. We emphasize that in addition to a remeasurement of the mass-12 spectra, measurements of the $\beta$-spectra for other cases in which the analog electromagnetic form factor is known, or can be measured, are very desirable.

Certain angular correlations, such as $\beta\gamma$ or $\beta\alpha$, have also been suggested as a way to measure weak magnetism. However, these correlations determine a combination of the weak magnetism and pseudo-tensor form factors, $b - d$, and without a clear understanding of the $d$-term, which entails the possible existence of second class currents, these measurements are necessarily ambiguous. An exception to this statement is the energy dependence of the electron-neutrino correlation, which depends on $b$ but not on $d$, but the possible cases here are few and difficult. In the following, we therefore limit our discussion to measurements of the shapes of momentum spectra.

In tables 10 and 11 we summarize nuclear beta transitions for which the analog isovector $b_\gamma$ term is already known. The tables are not intended to be exhaustive nor do certain of the entries derive from the most up-to-date data. Our intention is merely to make a rough evaluation of the possibilities.

Table 10 gives data for the $T = \frac{1}{2}$ mirror decays and tables 11 summarizes certain $T = 1$ to $T = 0$ decays. Cols. 1 and 2 specify the nuclei and spin sequence. The third column gives the experimental magnetic moments for the $T = \frac{1}{2}$ nuclei and the analog M1 width for the $T = 1$ to $T = 0$ decays. For the $T = 1$ to $T = 0$ decays, we calculate $b$, using eq. (19) whereas for the $T = \frac{1}{2}$ nuclei, we use

$$b = A \sqrt{\frac{J + 1}{J} \mu_\nu}, \quad (29)$$

in which $A$ is the mass number, $J$ is the spin, and $\mu_\nu$ is the isovector moment. From the $Ft$ values we calculate the values of $c$ using the approximate expression [see eq. (21)]

$$a^2 + c^2 \approx 2Ft(\text{Fermi})/Ft. \quad (30)$$

We omit several analog $\beta$- and $\gamma$-decays since many are weak $\beta$-branches or otherwise difficult cases. For a comprehensive comparison of analog decays, see ref. 35).
In col. 7 we tabulate the quantity \( b/Ac \) and observe that there are deviations of this from the neutron value. This variation is in part associated with the fact that, in the impulse approximation, the \( b \)-term has an orbital as well as spin component, whereas \( c \) is simply spin.

In col. 8 we give that part of the slope of the shape factor which is due to the \( c^2 \) and \( cb \) terms. That is, we use the approximate expression

\[
(dS/dE)_{abc} \approx \frac{2}{3M} \frac{3a^2 + 5c^2 + 2cb}{a^2 + c^2},
\]

where \( a \), the Fermi form factor, is unity for the \( T = \frac{1}{2} \) decays and zero for the others. This is our main result and we see that the values range from essentially zero for the \( ^{35}\text{Ar} \) decay to over 1 \( \% \) per MeV for the \( A = 8 \) and \( A = 20 \) decays.

In order to emphasize the importance of Coulomb effects, we also tabulate in col. 8 the main Coulomb correction for the point charge Fermi function, which is [cf. eq. (A.16)]

\[
(dS/dE)_{EM} \approx \frac{2a}{15} aZR.
\]

The total slope, given by the sum of eqs. (31) and (32) is tabulated in col. 9. Note that for the heavy nuclei the Coulomb term is a significant part of the total effect and thus it must be accurately known if \( b \) is to be determined with certainty.

Since our analysis of electromagnetic effects is of necessity imperfect, we suggest that future experiments be confined to low \( Z \) where small errors in calculated electromagnetic corrections will not mock up a sizeable weak magnetism term. Note that even the calculated corrections are subject to some uncertainty as they depend upon \( c_2/c_1 \). There is at present no reliable way to get an experimental handle on this quantity. For our analysis of the \( A = 12 \) data we have assumed, following the impulse approximation, that

\[
c(q^2)/c(0) \approx b(q^2)/b(0),
\]

and have obtained the right hand quantity from inelastic electron scattering data using CVC, which our analysis calls into question. This problem should be kept in mind, although such a relation is in reasonable agreement with the muon capture data on \(^{12}\text{C} \). An alternate approach is to use a strict impulse approximation, which gives

\[
\frac{c_2}{c_1} \approx \frac{1}{6} \frac{\beta||\tau_1\sigma r^2||\alpha\rangle}{\beta||\tau_1\sigma||\alpha\rangle} + \frac{1}{6\sqrt{10}} \frac{\beta||\tau_1\sigma||\alpha\rangle}{\beta||\tau_1\sigma||\alpha\rangle}.
\]

This value for \( A = 12 \) is 1.0 fm\(^2\) and is somewhat smaller than that obtained from the inelastic electron scattering. Such an ambiguity is difficult to avoid with present knowledge and gives a degree of model dependence to electromagnetic corrections, which reinforces our admonition against large-\( Z \) experiments if the objective is to measure \( b \). On the other hand, it may be interesting to test the theoretical estimates
<table>
<thead>
<tr>
<th>Decay</th>
<th>J(^*)</th>
<th>(\mu_i \mu_t ) ((\mu_{th}))</th>
<th>(b_0)</th>
<th>(F_{T'}) ((\text{sec}))</th>
<th>(c)</th>
<th>(b_0/Ac)</th>
<th>((dS/dE)<em>{abc}) ((dS/dE)</em>{ax})</th>
<th>((dS/dE)_{bc}) ((% \text{MeV}^{-1}))</th>
<th>(T_0) ((\text{MeV}))</th>
<th>(T_0(dS/dE)) ((%)</th>
</tr>
</thead>
</table>
| \(n \rightarrow p\) | \(
{1\over 2}^+\) | \(-1.913\) \((+2.793)\) | \(-8.15\) | \(1081(16)\) \(^4\) | \(-2.17\) | \(+3.76\) | \(+0.771\) \((+0.764)\) | \(0.78\) \((+0.60)\) |
| \(^3H \rightarrow ^3He\) | \(
{1\over 2}^+\) | \(-2.128\) \((+2.979)\) | \(+26.5\) | \(1143(3)\) \(^4\) | \(+2.10\) | \(+4.21\) | \(+0.602\) \((+0.582)\) | \(0.019\) \((+0.011)\) |
| \(^{11}C \rightarrow ^{11}B\) | \(
{3\over 2}^-\) | \(-1.03\) \((+2.689)\) | \(-52.8\) | \(3958(24)\) | \(-0.748\) | \(+6.42\) | \(-0.306\) \((+0.079)\) | \(-0.227\) \((+0.096)\) |
| \(^{13}N \rightarrow ^{13}C\) | \(
{1\over 2}^-\) | \(-0.322\) \((+0.702)\) | \(-23.1\) | \(4673(17)\) | \(-0.567\) | \(+3.13\) | \(-0.090\) \((+0.100)\) | \(+0.010\) \((1.20)\) |
| \(^{15}O \rightarrow ^{15}N\) | \(\{1\over 2\}^-\) | \(+0.719\) \((+0.283)\) | \(+26.0\) | \(4390(9)\) | \(+0.638\) | \(+2.72\) | \(-0.096\) \((+0.123)\) | \(+0.027\) \((1.73)\) |
| \(^{17}F \rightarrow ^{17}O\) | \(\{3\over 2\}^+\) | \(+4.722\) \((-1.894)\) | \(+133.1\) | \(2293(11)\) | \(+1.30\) | \(+6.02\) | \(-0.524\) \((+0.146)\) | \(-0.378\) \((1.74)\) |
| \(^{19}Ne \rightarrow ^{19}F\) | \(\{1\over 2\}^+\) | \(-1.887\) \((+2.629)\) | \(-148.6\) | \(1732(5)\) | \(-1.60\) | \(+4.89\) | \(-0.487\) \((+0.171)\) | \(-0.316\) \((2.22)\) |

**Table 10**

Predicted shape factors for the \(T = \frac{1}{2}\) mirror decays
<table>
<thead>
<tr>
<th>Reaction</th>
<th>Spin</th>
<th>Magnetic Moment</th>
<th>Energy (keV)</th>
<th>Q Value (keV)</th>
<th>R Value</th>
<th>Interaction Constant</th>
<th>Interaction Constant</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{21}$Na $\rightarrow$ $^{21}$Ne</td>
<td>$\frac{3}{2}^-$</td>
<td>$^{+}2.386$</td>
<td>$-0.662$</td>
<td>4036(79)</td>
<td>+0.728</td>
<td>+5.41</td>
<td>$-0.256$</td>
</tr>
<tr>
<td>$^{25}$Al $\rightarrow$ $^{25}$Mg</td>
<td>$\frac{5}{2}^+$</td>
<td>$^{+}3.646$</td>
<td>$-0.855$</td>
<td>3589(8)</td>
<td>+0.849</td>
<td>+6.27</td>
<td>$-0.365$</td>
</tr>
<tr>
<td>$^{29}$P $\rightarrow$ $^{29}$Si</td>
<td>$\frac{1}{2}^+$</td>
<td>$^{+}1.235$</td>
<td>$-0.555$</td>
<td>4866(41)</td>
<td>+0.519</td>
<td>+5.97</td>
<td>$-0.173$</td>
</tr>
<tr>
<td>$^{35}$Ar $\rightarrow$ $^{35}$Cl</td>
<td>$\frac{1}{2}^+$</td>
<td>$^{+}0.632$</td>
<td>+0.823</td>
<td>5648(19)</td>
<td>-0.306</td>
<td>+0.81</td>
<td>$-0.003$</td>
</tr>
<tr>
<td>$^{37}$K $\rightarrow$ $^{37}$Ar</td>
<td>$\frac{3}{2}^+$</td>
<td>$^{+}0.203$</td>
<td>$^{+}0.95 \pm 0.2$</td>
<td>4589(77)</td>
<td>-0.588</td>
<td>+1.64</td>
<td>$-0.054$</td>
</tr>
<tr>
<td>$^{39}$Ca $\rightarrow$ $^{39}$K</td>
<td>$\frac{3}{2}^+$</td>
<td>$^{+}1.022$</td>
<td>$^{+}0.391$</td>
<td>4278(25)</td>
<td>+0.666</td>
<td>+1.22</td>
<td>$-0.047$</td>
</tr>
<tr>
<td>$^{41}$Sc $\rightarrow$ $^{41}$Ca</td>
<td>$\frac{1}{2}^-$</td>
<td>$^{+}5.43$</td>
<td>$^{+}1.595$</td>
<td>2827(36)</td>
<td>+1.09</td>
<td>+7.31</td>
<td>$-0.562$</td>
</tr>
</tbody>
</table>

*) Except where noted otherwise all magnetic moments are taken from ref. 36).

b) Ref. 38).

*) Ref. 39).

d) Ref. 40).

*) Ref. 41).

r) The $F_I$ values are from ref. 42), except where noted.

b) Ref. 43).

r) Ref. 44).
### Table 11

Predicted shape factors for the $T = 1 \rightarrow T = 0$ decays

<table>
<thead>
<tr>
<th>Decay</th>
<th>$J_i \rightarrow J_f$</th>
<th>$\Gamma_{M1}$ (eV)</th>
<th>Ref.</th>
<th>$b_i$</th>
<th>$F_i$ (sec)</th>
<th>Ref.</th>
<th>$c$</th>
<th>$b_i/A_c$</th>
<th>$(dS/dE)_{h^+}$ (MeV)</th>
<th>$(dS/dE)_{h^-}$ (MeV)</th>
<th>$T_0$ (MeV)</th>
<th>$T_0(dS/dE)$ ($''_0$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^6\text{He} \rightarrow ^6\text{Li}$</td>
<td>$0^+ \rightarrow 1^+$</td>
<td>8.41 $\pm$ 0.25</td>
<td>$^45)$</td>
<td>69.0 $\pm$ 1.0</td>
<td>808 $\pm$ 2</td>
<td>$^42)$</td>
<td>2.75</td>
<td>4.18</td>
<td>+0.66</td>
<td>+0.62</td>
<td>3.5</td>
<td>+2.2</td>
</tr>
<tr>
<td>$^8\text{B} \rightarrow ^8\text{Be}$</td>
<td>$2^+ \rightarrow 2^+$</td>
<td>4.8 $\pm$ 0.7</td>
<td>$^46)$</td>
<td>$\approx 8.9$</td>
<td>$4.37 \times 10^5$</td>
<td>$\approx 0.119$</td>
<td>$\mp 1.32$</td>
<td>14</td>
<td>-17</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$^8\text{Li} \rightarrow ^8\text{Be}$</td>
<td>$1^+ \rightarrow 0^+$</td>
<td>1.32 $\times 10^4$</td>
<td>0.72</td>
<td>3.92</td>
<td>$\mp 0.56$</td>
<td>$\mp 0.46$</td>
<td>13</td>
<td>+16</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$^{12}\text{N} \rightarrow ^{12}\text{C}$</td>
<td>$1^+ \rightarrow 0^+$</td>
<td>37.0 $\pm$ 1.1</td>
<td>$^5)$</td>
<td>34.1 $\pm$ 1.1</td>
<td>$1.8 \times 10^4$</td>
<td>0.73</td>
<td>3.92</td>
<td>$\mp 0.56$</td>
<td>$\mp 0.10$</td>
<td>13.4</td>
<td>+6.2</td>
<td></td>
</tr>
<tr>
<td>$^{14}\text{O} \rightarrow ^{14}\text{N}$</td>
<td>$0^+ \rightarrow 1^+$</td>
<td>(7.6 $\pm$ 0.5) $\times 10^{-3}$</td>
<td>$^47)$</td>
<td>9.25</td>
<td>$1.9 \times 10^7$</td>
<td>$1.8 \times 10^{-2}$</td>
<td>37</td>
<td>$-5.3$</td>
<td>$\mp 0.1$</td>
<td>$\mp 38.7$</td>
<td>4.1</td>
<td>-21</td>
</tr>
<tr>
<td>$^{14}\text{C} \rightarrow ^{14}\text{N}$</td>
<td>$0^+ \rightarrow 1^+$</td>
<td>(7.6 $\pm$ 0.5) $\times 10^{-3}$</td>
<td>$^47)$</td>
<td>9.25</td>
<td>$1.9 \times 10^7$</td>
<td>$1.8 \times 10^{-2}$</td>
<td>37</td>
<td>$-5.3$</td>
<td>$\mp 0.1$</td>
<td>$\mp 38.7$</td>
<td>4.1</td>
<td>-21</td>
</tr>
<tr>
<td>$^{18}\text{Ne} \rightarrow ^{18}\text{F}$</td>
<td>$0^+ \rightarrow 1^+$</td>
<td>$\approx 0.16$</td>
<td>$^48)$</td>
<td>184</td>
<td>1240 $^48)$</td>
<td>2.2</td>
<td>4.57</td>
<td>+0.66</td>
<td>+0.49</td>
<td>3.4</td>
<td>+1.7</td>
<td></td>
</tr>
<tr>
<td>$^{18}\text{F} \rightarrow ^{18}\text{O}$</td>
<td>$1^+ \rightarrow 0^+$</td>
<td>9.79 $\times 10^4$</td>
<td>$^50)$</td>
<td>0.251</td>
<td>$9.40 \times 10^4$</td>
<td>$51^)$</td>
<td>0.256</td>
<td>8.79</td>
<td>$\mp 1.26$</td>
<td>$\mp 1.07$</td>
<td>11.2</td>
<td>-12</td>
</tr>
<tr>
<td>$^{20}\text{Na} \rightarrow ^{20}\text{Ne}$</td>
<td>$2^+ \rightarrow 2^+$</td>
<td>4.5 $\pm$ 0.5</td>
<td>$^48)$</td>
<td>44.5 $\pm$ 2.5</td>
<td>$9.40 \times 10^4$</td>
<td>$51^)$</td>
<td>0.256</td>
<td>8.79</td>
<td>$\mp 1.26$</td>
<td>$\mp 1.07$</td>
<td>11.2</td>
<td>-12</td>
</tr>
<tr>
<td>$^{20}\text{F} \rightarrow ^{20}\text{Ne}$</td>
<td>$2^+ \rightarrow 2^+$</td>
<td>4.5 $\pm$ 0.5</td>
<td>$^48)$</td>
<td>44.5 $\pm$ 2.5</td>
<td>$9.40 \times 10^4$</td>
<td>$51^)$</td>
<td>0.256</td>
<td>8.79</td>
<td>$\mp 1.26$</td>
<td>$\mp 1.07$</td>
<td>11.2</td>
<td>-12</td>
</tr>
<tr>
<td>$^{26}\text{Si} \rightarrow ^{26}\text{Al}(1.06)$</td>
<td>$0^+ \rightarrow 1^+$</td>
<td>0.018 $\pm$ 0.003</td>
<td>$^44)$</td>
<td>124</td>
<td>3390 $^44)$</td>
<td>1.35</td>
<td>3.5</td>
<td>+0.27</td>
<td>-0.23</td>
<td>3.0</td>
<td>-0.69</td>
<td></td>
</tr>
</tbody>
</table>
of the Coulomb term and the $^{35}$Ar spectrum appears to be interesting for this purpose. A better test of the Coulomb corrections, however, would be to measure the shapes of the spectra of the $0^+ \rightarrow 0^+$ pure Fermi decays. In this case, $b$ is exactly zero since only tensor operators with zero rank contribute to the decay. The decay of $^{54}$Co with $Z = 27$ and $E_0 \approx 7.2$ MeV would be particularly interesting in this regard.

We conclude by commenting on a few of the decays listed in tables 10 and 11. The neutron decay is, of course, a most interesting measurement but even though the intensity of cold neutron beams is sufficient, new spectrometer methods are needed before a precise measurement of the spectrum is possible. Considering the useful range of the energy spectrum, the size of the slope due to $b$, and the importance of minimizing the Coulomb corrections, we judge the best cases to be $^6$He, $^{12}$B-$^{12}$N, $^{17}$F, $^{19}$Ne and $^{20}$F-$^{20}$Na.

The $^6$He $e^-$ decay is very attractive in that it has only one decay branch, reasonably high energy, and low $Z$. Unfortunately, the mirror decay of $^6$Be is inaccessible since this nucleus is unbound.

The $A = 8$ decays are interesting but the unbound broad final state in $^8$Be adds a complication since one must measure the spectrum as a function of the excitation of the final state.

A measurement of $b$ for the $^{19}$Ne decay is very attractive since this is a decay within an isospin multiplet. This means that $b$ is solely first class, that is, $b = b^1$. The spectrum measurement would complement also the $\beta$-spin correlation experiment 37) which determines $b - d$.

The mass-20 decays have a large predicted slope and even though the end-point energies are lower than those of the mass-12 system, the useful part of the spectrum is comparable. With their longer lifetimes and simpler decay schemes, the $A = 20$ decays are thus in some ways more favorable than the mass-12 transitions. Again the spectrum measurements should provide a useful complement to on-going measurements of the $\beta\gamma$ correlation which determine the quantity $b - d$.

The $^{14}$O decay deserves special comment. Sindu and Gerhart 52) have measured the spectrum shape for the $0^+ \rightarrow 1^+$ transition and found a large slope, $dS/dE \approx -11\%$ MeV$^{-1}$. This is about twice as large as the prediction given in table 11 and thus appears to be in disagreement with the CVC theory. We suggest that this violation of CVC is probably due to the electromagnetic interaction which in this case is very much amplified, owing to the hindrance of the decays.

The $^6$He, $^{13}$N and $^{18}$F shape factors have been measured 9) but the accuracies are inadequate to observe the predicted effects. Finally we note that the shape factors of the $^{32}$P($1^+ \rightarrow 0^+$, log $ft = 7.9$) and the $^{41}$Ar($\frac{3}{2}^- \rightarrow \frac{5}{2}$, log $ft = 5.0$) electron decays both show a negative slope 9) which is opposite to what one naively expects. Unfortunately, the analog M1 decay rates have not been measured.
Appendix

In this section we examine a general method for evaluation of Coulomb corrections to allowed $\beta$-decay. It has been shown by Armstrong and Kim that the proper generalization of the $Z = 0$ weak amplitude

$$G \cos \theta_C \bar{u}(p)\gamma_\mu(1 + \gamma_5)\nu(l)\langle \beta_{p_1}|V^\mu + A^\mu|x_0\rangle,$$

(A.1)

to the $Z \neq 0$ case is given by

$$G \cos \theta_C \int \frac{d^3r}{(2\pi)^3} \Psi_\mu(r, p)\gamma_\mu(1 + \gamma_5)\nu(l) \int \frac{d^3s}{(2\pi)^3} e^{i r \cdot s} \langle \beta_{p_2 + p_1}|V^\mu + A^\mu|x_0\rangle.$$

(A.2)

where $\Psi(r, p)$ is a solution to the Dirac equation in the presence of the nuclear Coulomb potential which reduces as $Z \to 0$ to $\bar{u}(p) \exp (-ip \cdot r)$. Defining

$$\Psi(r, p) = \left(\frac{(2\pi)^3}{m_p}\right)^{\frac{1}{2}} \sum_{\kappa\mu} \bar{C}_{\mu - \rho, \mu} Y_{l, i}^\rho Y_{l - \rho, \mu}^\rho \psi_{\kappa\mu}(r),$$

(A.3)

where

$$\psi_{\kappa\mu}(r) = \left(\frac{g_{\kappa}(r)\chi_{\kappa, \mu}(\hat{r})}{if_{\kappa}(r)\chi_{\kappa, \mu}(\hat{r})}\right), \quad \gamma = \sqrt{\kappa^2 - \gamma^2 Z^2},$$

(A.4)

$$\nu = \frac{\alpha Z E}{p}, \quad \exp(2i\eta_\kappa) = \frac{-\kappa + i\alpha Z m/p}{\gamma + i\nu},$$

we can write, keeping leading $j = \frac{1}{2}$ and $\frac{3}{2}$ terms

$$\psi(r, p) \approx N(W + X\gamma^0 + Y\gamma \cdot \hat{r} + Z\gamma \cdot \dot{r}^0)\bar{u}(p),$$

(A.5)

with

$$N = \frac{1}{4\pi} \left(\frac{2\pi}{m_p}\right)^{\frac{1}{3}} \exp \left[-i\left[\frac{1}{2}\pi(1 - \gamma) + \eta_\mu - \arg \Gamma(\gamma + iv)\right]\right],$$

(A.6)

$$W = \frac{1}{2} \left(\frac{2m}{E + m}\right)^{\frac{1}{4}} \left\{ \frac{g_{-1}(r) + 3i\hat{r} \cdot \hat{p} e^{-i\delta_2} g_{-2}(r) + \frac{E + m}{p} (e^{-i\delta_1} f_1(r) + 3i\hat{r} \cdot \hat{p} e^{-i\delta_3} f_2(r))}{E + m} \right\}$$

(A.7)

$$\equiv W_1 + W_2 \hat{r} \cdot \hat{p},$$

$$X = \frac{1}{2} \left(\frac{2m}{E + m}\right)^{\frac{1}{4}} \left\{ \frac{g_{-1}(r) + 3i\hat{r} \cdot \hat{p} e^{-i\delta_2} g_{-2}(r) - \frac{E + m}{p} (e^{-i\delta_1} f_1(r) + 3i\hat{r} \cdot \hat{p} e^{-i\delta_3} f_2(r))}{E + m} \right\}$$

$$\equiv X_1 + X_2 \hat{r} \cdot \hat{p},$$

$$Y = \frac{i}{2} \left(\frac{2m}{E + m}\right)^{\frac{1}{4}} \left\{ \frac{f_{-1}(r) + e^{-i\delta_3} f_2(r) + \frac{E + m}{p} (e^{-i\delta_1} g_1(r) - e^{-i\delta_2} g_{-2}(r))}{E + m} \right\},$$

(A.8)
WEAK MAGNETISM

\[
Z = \frac{i}{2} \left( \frac{2m}{E+m} \right)^{\frac{1}{2}} \left[ f_{-1}(r) + e^{-i\theta} f_{2}(r) - \frac{E+m}{p} (e^{-i\theta} g_{1}(r) - e^{-i\theta} g_{-2}(r)) \right]
\]

\[
\delta_1 = \frac{1}{2} \pi + \eta_1 - \eta_{-1}, \quad \delta_2 = \frac{1}{2} \pi + \eta_{-2} - \eta_{-1}, \quad \delta_3 = \pi + \eta_2 - \eta_{-1}.
\]

For the sake of sanity keep only the leading Gamow-Teller term

\[
\langle \beta_{\mu} | V^\mu + A^\mu | \alpha_0 \rangle \approx -g^{\mu \nu} \langle E_0^2 - p_2^2 \rangle C_{f_1;1;j}^{M_1;M},
\]

and define the weak charge density

\[
\rho(r) = \int \frac{d^3s}{(2\pi)^3} e^{i\theta \cdot s} \frac{c(E_0^2 - (q + p - s)^2)}{c(0)}.
\]

Then the Coulomb corrections to the allowed spectrum can be written as

\[
d\lambda = \frac{4}{(2\pi)^3} G^2 \cos^2 \theta c(E_0 - E)^2 p E \delta h_1(E) \left[ |A|^2 + |B|^2 + |C|^2 + |D|^2 \right.
\]

\[
+ \frac{2}{9} \frac{p}{E} \Re (A^* D + B^* C) + 2 \frac{m_s}{E} \Re (A^* B + C^* D + \frac{1}{3} A^* C + \frac{1}{3} B^* D)
\]

\[
- \frac{2}{9} \frac{p}{E} \Re (A^* F - B^* G + 3F^* D - 3G^* C)
\]

where

\[
A = \int d^3r W_1^*(r)e^{-i \cdot r} N^* \rho(r), \quad B = \int d^3r X_1^*(r)e^{-i \cdot r} N^* \rho(r),
\]

\[
C = \int d^3r Y_1^*(r)e^{-i \cdot r} \dot{r} \cdot \dot{r} \rho(r), \quad D = \int d^3r Z_1^*(r)e^{-i \cdot r} \dot{r} \cdot \dot{r} \rho(r),
\]

\[
F = \int d^3r W_2^*(r)e^{-i \cdot r} \dot{r} \cdot \dot{r} \rho(r), \quad G = \int d^3r X_2^*(r)e^{-i \cdot r} \dot{r} \cdot \dot{r} \rho(r).
\]

As mentioned in the text, the problem is in principle done at this stage, as the integrals can be performed numerically. However, it is convenient to have simple analytic expressions useful for \(Z \alpha \ll 1\). Note that if we disregard the \(q^2\) dependence of \(c\) (i.e., \(\rho(r) = \delta^3(r)\)) then \(C = D = F = G = 0\) and

\[
|A|^2 + |B|^2 = \left. F_{\text{Behrens-Jaencke}}(Z, E) \right|.
\]

That is, the Coulomb correction is simply the Behrens-Jaencke Fermi function with no modification of the spectral function \(h_1(E)\). On the other hand writing

\[
c(q^2) = c_1 + c_2 q^2 + \ldots,
\]

as in eq. (5) we find

\[
\rho(r) = \left( 1 + \frac{c_2}{c_1} \left( E_0^2 + \nabla^2 \right) \right) \delta^3(r),
\]
\[ A = \left[ F_{\text{Behrens-Jänecke}}(Z, E) \right]^4 \left( 1 + \frac{c_2}{c_1} \left[ 2EE_0 - 2E^2 + m^2 - 3Z\alpha \frac{E}{R} - \frac{9}{4} \left( \frac{Z\alpha}{R} \right)^2 \right] \right) + \ldots, \]

(A.14)

\[ B \approx C \approx G \approx 0, \]

\[ D = \left[ F_{\text{Behrens-Jänecke}}(Z, E) \right]^4 \left( 2Z\alpha \frac{E_0 - E}{R} \frac{c_2}{c_1} \right) + \ldots, \]

(A.15)

\[ F = \left[ F_{\text{Behrens-Jänecke}}(Z, E) \right]^4 \left( -2p(E_0 - E) \frac{c_2}{c_1} \right) + \ldots, \]

which yields the spectrum indicated in eqs. (10) and (13), i.e. the Behrens-Jänecke Fermi function with a minor modification of the spectral function \( h_1(E) \). Note also that if we use \( c_2/c_1 = \frac{1}{10} R^2 \) corresponding to a uniform charge density, the Coulomb correction to the spectral function becomes

\[ F_{\text{Behrens-Jänecke}}(Z, E)[1 \pm \frac{2}{15} Z\alpha E_0 R + \frac{11}{15} Z\alpha ER \mp \frac{9}{20} (Z\alpha)^2] \]

\[ \approx F_0(Z, E)[1 \pm \frac{2}{15} Z\alpha E_0 R + \frac{24}{15} Z\alpha ER - \frac{9}{20} (Z\alpha)^2], \]

(A.16)

in good agreement with previous calculations due to Huffaker and Laird 16) and Armstrong and Kim 13).
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