Semileptonic Hyperon Decays and Cabibbo-Kobayashi-Maskawa Unitarity

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Using a technique that is not subject to first-order SU(3) symmetry breaking effects, we determine the $V_{us}$ element of the Cabibbo-Kobayashi-Maskawa matrix from data on semileptonic hyperon decays. We obtain $V_{us} = 0.2250(27)$, where the quoted uncertainty is purely experimental. This value is of similar experimental precision to the one derived from $K_{l3}$, but it is higher and thus in better agreement with the unitarity requirement, $|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$. An overall fit, including the axial contributions and neglecting SU(3) breaking corrections, yields $F + D = 1.2670 \pm 0.0035$ and $F - D = -0.341 \pm 0.016$ with $\chi^2 = 2.96/3$ degrees of freedom.

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The determination of the elements of the Cabibbo-Kobayashi-Maskawa (CKM) matrix [1,2] is one of the main ingredients for evaluating the solidity of the standard model of elementary particles. This is a vast subject of the parameters that can be measured from hyperon beta decays [9,10], and degrades the precision. If on the contrary, as we do here, one introduces adjustable SU(3) breaking parameters, as is done in the model-dependent radiative corrections [11] are applied, this increases the number of degrees of freedom (DOF) and degrades the precision. If on the contrary, as we do here, one focuses the analysis on the form factors, treating the rates and $g_1/f_1$ as the basic experimental data, one has direct access to the $f_1$ form factor for each decay, and this in turn allows for a redundant determination of $V_{us}$. The consistency of the values of $V_{us}$ determined from the different decays is a first confirmation of the overall consistency of the model. A more detailed discussion may be found in the Annual Reviews of Nuclear and Particle Sciences [12].

In 1964, Ademollo and Gatto proved [13] that there is no first-order correction to the vector form factor, $\Delta f_1(0) = 0$. This is an important result: since experiments can measure $V_{us} f_1(0)$, knowing the value of $f_1(0)$ in $\Delta S = 1$ decays is essential for determining $V_{us}$.

The Ademollo-Gatto theorem suggests an analytic approach to the available data that first examines the vector form factor $f_1$ because it is not subject to first-order SU(3) symmetry breaking effects. An elegant way to do this is to use the measured value of $g_1/f_1$ along with the predicted values of $f_1$ and $f_2$ to extract a $V_{us}$ value from the decay rate for each decay. If the theory is correct, these should coincide within errors and could be combined to obtain a best value of $V_{us}$. This consistency of the $V_{us}$ values obtained from different decays then indicates the success of the Cabibbo model. A similar approach appears to have been taken in Ref. [14].

Four hyperon beta decays have sufficient data to perform this analysis: $\Lambda \rightarrow p e^- \bar{\nu}, \Sigma^- \rightarrow n e^- \bar{\nu}, \Xi^- \rightarrow \Lambda e^- \bar{\nu},$ and $\Xi^0 \rightarrow \Sigma^+ e^- \bar{\nu}$ [9]. Table I shows the results for them. In this analysis, both model-independent and model-dependent radiative corrections [11] are applied, and the $q^2$ variation of $f_1$ and $g_1$ is included. Also SU(3) values of $g_2 = 0$ and $f_2$ are used along with the numerical rate expressions tabulated in Ref. [11]. We have not, however, included SU(3) breaking corrections to the $f_1$ form factor, which will be discussed in the next section. The
stated $V_{us}$ errors are purely experimental, coming from experimental uncertainties in the hyperon lifetimes, branching ratios, and form factor ratios.

The four values are clearly consistent ($\chi^2 = 2.26/3$ DOF) with the combined value of $V_{us} = 0.2250 \pm 0.0027$. This value is nearly as precise as that obtained from kaon decay ($V_{us} = 0.2196 \pm 0.0023$) and, as observed in previous analyses [15–17], is somewhat larger. In combination with $V_{us} = 0.9740 \pm 0.0005$ obtained from superallowed pure Fermi nuclear decays [8], the larger $V_{us}$ value from hyperon decays beautifully satisfies the unitarity constraint $|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$.

We will limit our discussion to the effects that are most relevant for the determination of $V_{us}$. Turning our attention first to SU(3) breaking corrections to the $f_1$ form factor, we find in the literature computations that use some version of the quark model, as in [18,19], or some version of chiral perturbation theory, as in [15,20,21].

The quark-model computations find that the $f_1$ form factors for the different $\Delta S = 1$ decays are reduced by a factor, the same for all decays, given as 0.987 in [18] and 0.975 in [19], a decrease, respectively, of 1.3% or 2.5%. This is a very reasonable result, the decrease arising from the mismatch of the wave functions of baryons containing different numbers of the heavier $s$ quarks.

Evaluations of $f_1$ in chiral perturbation theory range from small negative corrections in [20] to larger positive corrections in [15,21]. Positive corrections in $f_1$ for all hyperon beta decays cannot be excluded but are certainly not expected in view of an argument [22] according to which one expects a negative correction to $f_1$, at least in the $\Sigma^- \to n e^- \nu$ case. This result follows from the observation that the intermediate states that contribute to the positive second-order terms in the Ademollo and Gatto sum rule have, in this case, quantum numbers $S = -2$, $I = 3/2$; no resonant baryonic state is known with these quantum numbers. If we accept the hypothesis that the contribution of resonant hadronic states dominate, we can conclude that the correction to $f_1$ in $\Sigma^-$ beta decay should be negative. We note that this argument also applies to $K_{L1}$ decays, and that the corrections to these decays, computed with chiral perturbation theory, are, as expected, negative.

A modern revisitation of the quark-model computations will be feasible in the near future with the technologies of lattice QCD, and we would expect that a small negative correction would be obtained in quenched lattice QCD, an approximation that consists in neglecting components in the wave function of the baryons with extra quark-antiquark pairs. This is known to be an excellent approximation in low-energy hadron phenomenology [23].

Multiquark effects can be included in lattice QCD by forsaking the quenched approximation for a full simulation. Alternatively, one could resort to chiral perturbation theory to capture the major part of the multiquark contributions which will be dominated by virtual $\pi, K, \eta$ states. Early results of a similar strategy applied to the $K_{L3}$ decays [24] indicate that in that case a 1% determination of the $f_+(0)$ form factor is within reach, and we expect that a similar precision can be obtained in the case of hyperon decays. In the present situation, we consider it best not to include any SU(3) breaking corrections in our evaluation, nor to include an evaluation of a theoretical error. Our expectation that the corrections to $f_+(0)$ will be small and negative can only be substantiated by further work.

We next turn our attention to the possible effect of ignoring the $g_2$ form factor. In the absence of second class currents [25], the form factor $g_2$ can be seen to vanish in the SU(3) symmetry limit. The argument is very straightforward: the neutral currents $A^3_\alpha = \bar{q} \lambda^3 \gamma_\alpha \gamma_5 q$ and $A^8_\alpha = \tilde{q} \lambda^8 \gamma_\alpha \gamma_5 q$ that belong to the same octet as the weak axial-vector current are even under charge conjugation, so that their matrix elements cannot contain a weak-electricity term, which is $C$ odd. The vanishing of the weak electricity in the proton and neutron matrix elements of $A^3_\alpha, A^8_\alpha$ implies the vanishing of the $D$ and $F$ coefficients for $g_2(0)$, so that, in the SU(3) limit, the $g_2(0)$ form factor vanishes for any current in the octet.

In hyperon decays, a nonvanishing $g_2(0)$ form factor can arise from the breaking of SU(3) symmetry. Theoretical estimates [26] indicate a value for $g_2(0)/g_1(0)$ in the $-0.2$ to $-0.5$ range.

In determining the axial-vector form factor $g_1$ from the Dalitz Plot—or, equivalently, the electron-neutrino correlation—one is actually measuring $\bar{g}_1$, a linear combination of $g_1$ and $g_2$ ($\bar{g}_1 = g_1 - \delta g_2$ up to first order in $\delta = \Delta M/M$). This has already been noticed in past experiments and is well summarized in Gaillard and

### Table I Results from $V_{us}$ analysis using measured $g_1/f_1$ values.

<table>
<thead>
<tr>
<th>Decay Process</th>
<th>Rate ($\mu$sec$^{-1}$)</th>
<th>$g_1/f_1$</th>
<th>$V_{us}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Lambda \to p e^- \nu$</td>
<td>3.161(58)</td>
<td>0.718(15)</td>
<td>0.2224 ± 0.0034</td>
</tr>
<tr>
<td>$\Sigma^- \to n e^- \nu$</td>
<td>6.88(24)</td>
<td>-0.340(17)</td>
<td>0.2282 ± 0.0049</td>
</tr>
<tr>
<td>$\Xi^- \to \Lambda e^- \nu$</td>
<td>3.44(19)</td>
<td>0.25(5)</td>
<td>0.2367 ± 0.0099</td>
</tr>
<tr>
<td>$\Xi^0 \to \Sigma^+ e^- \nu$</td>
<td>0.876(71)</td>
<td>1.32(+0.22/−0.18)</td>
<td>0.209 ± 0.027</td>
</tr>
<tr>
<td>Combined</td>
<td></td>
<td></td>
<td>0.2250 ± 0.0027</td>
</tr>
</tbody>
</table>
and assumption of neglecting the case, we may conclude that making the conventional contributions. Quite possible to improve the present situation on the Vus uncertainties of the early experimental results. We also effects, seems to indicate that such effects were over-

peron beta decays as well as in theoretical and the experimental side. Indication that more work remains to be done both on the theoretical and the experimental side. 

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Note added.—The KTeV Collaboration has just announced a determination of $|V_{us}|$ based on neutral kaon semileptonic decay rates [30]. The result $|V_{us}| = 0.2252(\pm 0.0005_{\mathrm{KTeV}} \pm 0.009_{\mathrm{stat}})$ is in beautiful agreement with our hyperon-derived value.

We have given some indication that the trouble could arise from the $K_{l3}$ determination of $V_{us}$, and we would like to encourage further experimental work in this field [29].

The excellent agreement with the unitarity condition of our determination of $V_{us}$, which neglects SU(3) breaking effects, seems to indicate that such effects were overestimated in the past, probably as a consequence of the uncertainties of the early experimental results. We also find [12] that the $g_1$ form factor of the different decays, which is subject to first-order corrections, is well fitted by the $F, D$ parameters [1], with $F + D = 1.2670 \pm 0.0035$ and $F - D = -0.341 \pm 0.016$ with $\chi^2 = 2.96/3$ DOF.

The value of $V_{us}$ obtained from hyperon decays is of comparable experimental precision with that obtained from $K_{l3}$ decays and is in better agreement with the value of $\theta_C$ obtained from nuclear beta decay. While a discrepancy between $V_{us}$ and $V_{ud}$ could be seen as a portent of exciting new physics, a discrepancy between the two different determinations of $V_{us}$ can be taken only as an indication that more work remains to be done both on the theoretical and the experimental side.

On the theoretical side, renewed efforts are needed for the determination of $V_{us}$ breaking effects in hyperon beta decays as well as in $K_{l3}$ decays. While it is quite possible to improve the present situation on the quark-model front, the best hopes lie in lattice QCD simulations, perhaps combined with chiral perturbation theory for the evaluation of large-distance multiquark contributions.

Sauvage [17], Table 8. Therefore, in deriving $V_{us}^2 f_1^2$ (hence $V_{us}$) from the beta decay rate, there is in fact a small sensitivity to $g_2$. To first order, the rate is proportional to $V_{us}^2 f_1^2 + 3g_1^2 + 4g_1 g_2 = V_{us}^2 f_1^2 + 3g_1^2 + 2\delta g_1 g_2$. In fact, this is a second-order correction to the value of $V_{us}$, potentially of the same order of magnitude as the corrections to $f_1$.

Experiments that measure correlations with polarization—in addition to the electron-neutrino correlation—are sensitive to $g_2$. While the data are not yet sufficiently precise to yield good quantitative information, one may nevertheless look for trends. In polarized $\Sigma^+ \rightarrow p e^- \bar{\nu}$ [27], negative values of $g_2/f_1$ are clearly disfavored (a positive value is preferred by $1.5\sigma$). Since the same experiment unambiguously established that $g_1/f_1$ is negative, one concludes that allowing for nonvanishing $g_2$ would increase the derived value of $V_{us}^2 f_1^2$. In polarized $\Lambda \rightarrow p e^- \bar{\nu}$, the data favor [28] negative values of $g_2/f_1$ (by about $2\sigma$). In this decay, $g_1/f_1$ is positive so that, again, allowing for the presence of nonvanishing $g_2$ would increase the derived value of $V_{us}^2 f_1^2$. In either case, we may conclude that making the conventional assumption of neglecting the $g_2$ form factor tends to underestimate the derived value of $V_{us}$. A more quantitative conclusion must await more precise experiments. We consider it to be of the highest priority to determine the $g_2$ form factor (or a stringent limit on its value) in at least one of the hyperon decays, ideally in $\Lambda$ semileptonic decay, which at the moment seems to offer the single most precise determination of $V_{us}$.

The value of $V_{us}$ obtained from hyperon decays is

determination of $V_{us}$, as it involves the intricate and elegant relationships that the model predicts.

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[24] We are grateful to Guido Martinelli for a discussion on this point; see D. Bećirević et al., hep-ph/0403217.
[29] In fact, a recently reported result from the Brookhaven E865 Collaboration indicates a higher $K_{e3}$ decay rate than that previously used to determine $V_{us}$. See Brookhaven E865 Collaboration, A. Sher et al., Phys. Rev. Lett. 91, 261802 (2003).