ESTABLISHMENT OF CP VIOLATION IN B DECAYS

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Abstract  Until recently, CP violation had been observed only in kaon decays. In order to conclusively interpret CP violation and test whether it can be explained in the framework of the Cabibbo-Kobayashi-Maskawa matrix, two asymmetric B factories, KEK-B and PEP-II, were constructed in the past decade. Recent measurements with the two detectors at these B factories clearly established large CP violation in neutral B mesons. We review the results and outline the prospects for future measurements.

CONTENTS

1. INTRODUCTION ................................................... 354
2. ACCELERATORS AND DETECTORS .................................. 357
   2.1. Accelerators ............................................ 357
   2.2. Detectors .................................................... 359
3. RECONSTRUCTION OF B MESONS .................................. 360
   3.1. Reconstruction of Intermediate Meson States .............. 361
   3.2. Reconstruction of $B^0 \rightarrow J/\psi K^0_L$ .................. 365
   3.3. Reconstruction of $B^0 \rightarrow D^- \ell^+ \nu$ ................. 366
4. MEASUREMENT OF $\Delta t$ ............................................. 367
   4.1. $\Delta t$ Reconstruction .................................... 368
   4.2. $\Delta t$ Resolution Function ................................. 369
5. DETERMINATION OF THE $b$ FLAVOR OF THE ACCOMPANYING B MESON ........................................ 370
   5.1. Flavor-Tagging Method ...................................... 372
   5.2. Flavor-Tagging Performances ............................... 372
6. LIKELIHOOD FIT ................................................... 373
   6.1. Background Parameterization ............................... 375
   6.2. Direct CP Violation ....................................... 376
7. RESULTS .......................................................... 376
   7.1. Consistency Checks ......................................... 379
1. INTRODUCTION

CP violation has been a puzzle in particle physics since 1964, when it was discovered in neutral kaon decay by Cronin & Fitch (1). Interest was heightened by Sakharov’s observation (2) in 1967 that without CP violation, a universe that began as matter-antimatter symmetric could not have evolved into the asymmetric one that we now inhabit. In 1973, Kobayashi & Maskawa (KM) proposed a model that incorporates CP violation via an irreducible complex phase in the weak-interaction quark mixing matrix (3). The idea, presented at a time when only the u, d, and s quarks were known to exist, was remarkable because it extended Cabibbo’s quark mixing model from three to six quarks (4). The subsequent discoveries of the c, b, and t quarks, and the compatibility of the model with the CP-violating effects observed in the neutral K meson system, led to the incorporation of the KM mechanism into the standard model, even though it had not been conclusively proven experimentally.

Complex phases of decay amplitudes can be observed in processes in which several amplitudes with different phases contribute. This can occur in meson-antimeson (M-\overline{M}) systems with large mixing probabilities, when the decay of the M meson to a final state f can occur in two ways: either directly, \( M \rightarrow f \), or via mixing to the antimeson, \( M \rightarrow \overline{M} \rightarrow f \). The difference in phase of the two contributing amplitudes can be extracted from the time dependence of the decay rate.

Until recently, the kaon system was the only system in which CP violation had been observed, but unfortunately this observation cannot be easily interpreted in terms of standard-model parameters owing to large uncertainties associated with nonperturbative QCD effects. In 1983, the MAC and MARK-II collaborations at PEP (5) observed that the B-meson lifetime was \( O(10^{-12}) \) s, much longer than expected from the existing theoretical prejudices. In 1987, the ARGUS collaboration announced the surprising discovery of large \( B_d-\overline{B}_d \) mixing (6). As a result, it became experimentally feasible to test the proposal by Bigi, Carter, and Sanda (7) that there would be large asymmetries in hadronic B decays to CP eigenstates. This realization eventually led to the construction of two high-luminosity \( e^+e^-B \) factories.

The Cabibbo-Kobayashi-Maskawa (CKM) weak-interaction quark mixing matrix is defined as

\[
\begin{pmatrix}
V_{ud} & V_{us} & V_{ub} \\
V_{cd} & V_{cs} & V_{cb} \\
V_{td} & V_{ts} & V_{tb}
\end{pmatrix},
\]

where the nontrivial complex phases are assigned to the furthest off-diagonal elements, \( V_{ub} \) and \( V_{td} \). Unitarity of the CKM matrix implies that \( \Sigma_i V_{ij} V_{ik}^* = \delta_{jk} \), which gives the following relation involving \( V_{ub} \) and \( V_{td} \):

\[
V_{ud} V_{ub}^* + V_{cd} V_{cb}^* + V_{td} V_{tb}^* = 0.
\]
This expression can be visualized as a closed triangle in the complex plane, as shown in Figure 1.

The unitarity of the CKM matrix implies the existence of three measurable phases that are related to the interior angles of the unitarity triangle. In the "Ni-hongo" convention, these angles are denoted as

$$\phi_1 \equiv \arg \left( \frac{V_{cd} V_{cb}^*}{V_{td} V_{tb}^*} \right), \quad \phi_2 \equiv \arg \left( -\frac{V_{ud} V_{ub}^*}{V_{td} V_{tb}^*} \right), \quad \phi_3 \equiv \arg \left( -\frac{V_{cd} V_{cb}^*}{V_{ud} V_{ub}^*} \right).$$

These angles are also referred to as $\beta$, $\alpha$, and $\gamma$, respectively.

Measurements of CKM parameters can be translated into confidence regions for the location of the apex of the triangle. Figure 1 shows the corresponding constraints prior to the asymmetric $B$ factories (see for instance Reference 8 and references therein). Asymmetric $B$ factories allow unprecedented precision in the measurements of some of the angles of the triangle, which allow us to study and confirm the mechanism that was previously observed only in the kaon decays.

Asymmetric $B$ factories allow direct measurements of the interior angles of the unitary triangle in $B$ decays (for more complete pedagogical discussions of
The time-dependent asymmetry in the decay rate of $B_f$ eigenstate unitary triangle, $\eta$, where $|\lambda|=1$, mixing frequency, and $\tau_B$ is the lifetime. The complex parameter $\lambda = e^{-2\phi}/A = |\lambda| e^{2\phi + \phi_f}$ depends on the amplitudes for $B^0(A)$ and $\bar{B}^0(\bar{A})$ decay to the state $f$. If $f$ is a $CP$ eigenstate and only one amplitude contributes to the process, $|\lambda|=1$ and therefore Equation 4 reduces to

$$f_{\pm}(\Delta t) = \frac{e^{-|\Delta t|/\tau_B}}{2\tau_B(1 + |\lambda|^2)} \left\{ \frac{1 + |\lambda|^2}{2} \pm \left( \frac{1}{2} \cos(\Delta m_d \Delta t) - \frac{1}{2} \right) \right\}, \quad 4.$$ 

where the $+$ ($-$) sign refers to $B^{0}(\bar{B}^{0})$ mesons, $\Delta m_d$ is the neutral $B$ mesons’ mixing frequency, and $\tau_B$ is the lifetime. The complex parameter $\lambda = e^{-2\phi}/A = |\lambda| e^{2\phi + \phi_f}$ depends on the amplitudes for $B^0(A)$ and $\bar{B}^0(\bar{A})$ decay to the state $f$. If $f$ is a $CP$ eigenstate and only one amplitude contributes to the process, $|\lambda|=1$ and therefore Equation 4 reduces to

$$f_{\pm}(\Delta t) = \frac{e^{-|\Delta t|/\tau_B}}{4\tau_B(1 + |\lambda|^2)} \left[ 1 \pm \left( \frac{1}{2} \cos(\Delta m_d \Delta t) \right) \right]. \quad 5.$$ 

The time-dependent asymmetry in the decay rate of $B^0$ and $\bar{B}^0$ to a common $CP$ eigenstate $f$ determines the phase of $\lambda$, which is a function of the angles of the unitary triangle,

$$A_f(\Delta t) = \frac{N(B^0 \to f) - N(\bar{B}^0 \to f) \to f)}{N(B^0 \to f) + N(B^0 \to f)} \equiv -\eta_f \sin(2\phi_1 + \phi_f) \sin(\Delta m_d \Delta t), \quad 6.$$ 

where $\eta_f$ is the $CP$ eigenvalue of the state $f$. For decays $B \to X_{cc} K^0_s$ or $B \to X_{cc} K^0_s$, where $X_{cc}$ is any charmonium state, all amplitudes have the same phase and therefore $A_f(\Delta t) \equiv -\eta_f \sin(2\phi_1) \sin(\Delta m_d \Delta t)$ and, e.g., $\eta_f = -1$ for $f = X_{cc} K^0_s$ and $\eta_f = +1$ for $f = J/\psi K^0_s$.

Neutral $B$ mesons originating from the decay of the $\Upsilon(4S)$ resonance are produced in a coherent state and, in order to satisfy Bose-Einstein statistics, when one of the two $Bs$ decays, the other one must have opposite flavor. If the $b$-quark flavor of one of the two mesons is identified or “tagged,” Equations 4–6 apply. At the time of first decay, the flavors of the two $B$ mesons are opposite. The time interval $\Delta t$ between the decays $B \to f$ and the tagged $B$ may have either a positive or a negative sign, depending on which $B$ decays first. At the $\Upsilon(4S)$, the time-integrated asymmetry is zero, and thus it is necessary to measure $\Delta t$. Unfortunately, the distances traveled by $B$ mesons produced in the decay of a $\Upsilon(4S)$ at rest are too small ($\sim 25 \mu m$) to be resolved with present detector capabilities. Therefore, as first suggested by Oddone (10), it is necessary to boost the $\Upsilon(4S)$. This can be achieved in $e^+ e^-$ storage rings with beams of different energies. In such an asymmetric-energy storage ring, the $B$-meson decay distances are dilated to a measurable length.
In order to accomplish this experimental program, two high-luminosity $e^+e^-$ $B$ factories, PEP-II and KEKB, were constructed in the United States and Japan (respectively) and two detectors, BABAR and Belle, have begun collecting large samples of data (11, 12). At KEKB (PEP-II), the $e^-$ beam has an energy of 8 (9) GeV whereas the $e^+$ beam has an energy of 3.5 (3.1) GeV. The $\beta\gamma$ of the $\Upsilon(4S)$ is approximately 0.43 (0.56).

The $B$ factories were commissioned with remarkable speed starting in late 1998, and the experiments started operation the following year. The first results appeared in February 2001 (13). In the summer of 2001, these two experiments announced the observation of the first statistically significant signals for $CP$ violation outside of the kaon system (14, 15). In the summer of 2002, the $B$-factory experiments announced precise measurements in agreement with the standard-model expectations based on indirect determinations (16, 17).

In summary, the measurement of time-dependent $CP$ asymmetry requires the following:

1. A large sample of $\Upsilon(4S)$ decays into $B_0\bar{B}_0$ pairs, with the $\Upsilon(4S)$ frame boosted to dilate the $B$ mesons' decay length to a measurable distance. Section 2 describes the KEKB and PEP-II storage rings and the corresponding detectors, Belle and BABAR.

2. Efficient reconstruction of $B \to X_{c\bar{c}}K^0$ decays. As described in Section 3, this implies an accurate measurement of momenta and energies as well as efficient identification of leptons, photons, and $K^0_S$ and $K^0_L$ mesons.

3. A measurement of $\Delta t$ is derived from the measurement of $\Delta z$, the spatial distance between the decay vertices, as explained in Section 4.

4. The determination of the flavor of the accompanying $B$ (“tagging”). As discussed in Section 5, this requires the identification of electrons, muons, and charged kaons and the measurement of their charge.

2. ACCELERATORS AND DETECTORS

In order to satisfy the requirements for the measurement of $CP$ asymmetries, the design of the $e^+e^-$ $B$ factories and the experiments demanded substantial improvements in accelerator technology and, to a lesser degree, in detector technology (18).

2.1. Accelerators

The experimental requirement for a large data sample translates to the need for high luminosity. Table 1 summarizes the design parameters of the $B$-factory storage rings that achieve luminosities in excess of $3 \times 10^{33}/\text{cm}^2/\text{s}$ (19, 20). The properties of the individual bunches are not dramatically different from those achieved at conventional storage rings such as CESR. To obtain high luminosity, the asymmetric $B$ factories store a very large number of bunches in two rings. Complex feedback systems, similar to those in high-intensity light sources, are required to avoid instabilities involving coupling of bunches.
To minimize parasitic collisions between incoming and outgoing bunches, KEK-B (20) chose to collide the two beams at a small angle, whereas PEP-II (19) collides them head-on, takes advantage of the energy asymmetry, and employs magnetic separation. Originally there was some worry that coupled longitudinal and transverse oscillations of the beams (synchro-betatron oscillations) might be excited for a nonzero crossing angle, but this turned out not to be the case. The magnetic-separation approach gives rise to somewhat larger experimental backgrounds and imposes tighter constraints on the engineering of the interaction region.

KEK-B and PEP-II have achieved remarkable performance in a very short time. Within the first three years of operation, both had delivered more than 100 fb$^{-1}$. The peak luminosity for PEP-II is $6.5 \times 10^{33}/\text{cm}^2/\text{s}$; KEK-B has achieved $1.0 \times 10^{34}/\text{cm}^2/\text{s}$. However, some of the parameters used to achieve these luminosities differ from the design parameters in Table 1. The data analyses for the $CP$ violation parameter $\sin 2\beta$ (or $\sin 2\phi_1$) are based on the datasets recorded on the peak of the $\Upsilon(4S)$ resonance. For background studies, $\sim 10\%$ of the time data are recorded 40 MeV below resonance. The on-resonance datasets with integrated luminosities of 78 fb$^{-1}$ for Belle and 81 fb$^{-1}$ for BABAR correspond to 85 million and 88 million $B \overline{B}$ pairs, respectively.

### TABLE 1  Design parameters for B-factory accelerators

<table>
<thead>
<tr>
<th></th>
<th>KEK-B</th>
<th>PEP-II</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>LER</td>
<td>HER</td>
</tr>
<tr>
<td>Energy $E$ (GeV)</td>
<td>3.5</td>
<td>8.0</td>
</tr>
<tr>
<td>Luminosity $L$ (cm$^{-2}$s$^{-1}$)</td>
<td>$1 \times 10^{34}$</td>
<td>$3 \times 10^{33}$</td>
</tr>
<tr>
<td>Collision mode</td>
<td>$\pm 11mrad$</td>
<td>Head-on</td>
</tr>
<tr>
<td>Circumference $C$ (m)</td>
<td>3018</td>
<td>2199</td>
</tr>
<tr>
<td>Beta function $\beta^<em>_x/\beta^</em>_y$ (cm)</td>
<td>100/1</td>
<td>100/1</td>
</tr>
<tr>
<td>Tune shift $\xi_x/\xi_y$</td>
<td>0.05/0.05</td>
<td>0.03/0.03</td>
</tr>
<tr>
<td>Emittance $\epsilon_x/\epsilon_y$ (nm)</td>
<td>19/0.19</td>
<td>19/0.19</td>
</tr>
<tr>
<td>Energy spread $\sigma_E/E$ ($10^{-4}$)</td>
<td>7.7</td>
<td>7.2</td>
</tr>
<tr>
<td>Total current $I$ (A)</td>
<td>2.6</td>
<td>1.1</td>
</tr>
<tr>
<td>No. bunches $N_B$</td>
<td>5120</td>
<td>1658</td>
</tr>
<tr>
<td>Bunch spacing $S_B$ (m)</td>
<td>0.6</td>
<td>1.26</td>
</tr>
<tr>
<td>RF frequency $f_{RF}$ (MHz)</td>
<td>508</td>
<td>476</td>
</tr>
<tr>
<td>RF voltage $V_c$ (MV)</td>
<td>22</td>
<td>48</td>
</tr>
<tr>
<td>Cavity type</td>
<td>ARES</td>
<td>super</td>
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<tr>
<td>No. cavities $N_c$</td>
<td>28</td>
<td>60</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>20</td>
</tr>
</tbody>
</table>
2.2. Detectors

In order to satisfy the requirements discussed in Section 1, the measurement of CP violation requires accurate reconstruction of charged-particle trajectories and good identification of electrons, muons, charged kaons, photons, and $K^0_S$ and $K^0_L$ mesons.

As shown in Table 2, in contrast to the two storage rings, the two detectors are quite similar except for their particle-identification systems (11, 12).

The most challenging experimental requirement is the detection of the decay points of the short-lived $B$ mesons. Both experiments use double-sided silicon strips, three layers in Belle and five in BABAR, allowing full tracking of low-momentum tracks. To minimize the contribution of multiple scattering, these detectors are located at small radii close to the interaction point. For tracking outside the silicon detector, and the measurement of momentum, both experiments use conventional drift chambers with a helium-based gas mixture to minimize multiple scattering and synchrotron radiation backgrounds.

The other difficult requirement for the detectors is the separation of kaons from pions. At high momentum, this is needed to distinguish $\bar{B}^0 \rightarrow \pi^+\pi^-$ from $\bar{B}^0 \rightarrow K^-\pi^+$. At lower momenta, particle identification is essential for $B$ flavor tagging.

Two approaches to high-momentum particle identification have been implemented, both based on the use of Cerenkov radiation. At Belle, aerogel is used as a radiator. Blocks of aerogel are read out directly by fine-mesh phototubes that have high gain and operate reliably in a 1.5-Tesla magnetic field. Because the threshold momentum for pions in the aerogel is $\sim 1.5$ GeV/$c$, below this momentum $K/\pi$

<table>
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<tr>
<th>Component</th>
<th>BABAR</th>
<th>Belle</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vertex tracker SVT</td>
<td>5 layers</td>
<td>3 layers</td>
</tr>
<tr>
<td>SVT radius (cm)</td>
<td>3.2–14.4</td>
<td>3–5.8</td>
</tr>
<tr>
<td>CDC</td>
<td>40 layers</td>
<td>50 layers</td>
</tr>
<tr>
<td>CDC radius (cm)</td>
<td>24–80</td>
<td>9–86</td>
</tr>
<tr>
<td>Particle identification</td>
<td>DIRC+dE/dx</td>
<td>Aerogel/TOF+dE/dx</td>
</tr>
<tr>
<td>Electromagnetic calorimeter</td>
<td>CsI (Tl)</td>
<td>CsI (Tl)</td>
</tr>
<tr>
<td>Magnetic field</td>
<td>1.5 T</td>
<td>1.5 T</td>
</tr>
<tr>
<td>$K_L/\mu$ detector</td>
<td>RPC</td>
<td>RPC</td>
</tr>
<tr>
<td>Data transfer</td>
<td>Digital</td>
<td>Q-to-T into</td>
</tr>
<tr>
<td></td>
<td>Pipeline</td>
<td>multihit TDC</td>
</tr>
</tbody>
</table>

*Abbreviations: CDC, conventional drift chamber; RPC, resistive plate chamber; Q-to-T, charge to time conversion; TDC, time-to-digital converter."
separation is carried out using high-precision time-of-flight (TOF) scintillators with a resolution of 95 ps. The aerogel and TOF counter system is complemented by dE/dx measurements in the central drift chamber. The dE/dx system provides K/π separation below 0.7 GeV/c and above \( \sim 2.5 \text{ GeV}/c \) in the relativistic rise region.

At BABAR, Cerenkov light is produced in quartz bars and then transmitted by total internal reflection to the outside of the detector through a water tank to a large array of phototubes where the ring is imaged. The detector is called DIRC (Detector of Internally Reflected Cerenkov light). It provides particle identification for particles above 700 MeV/c. Additional particle identification is provided by dE/dx measurements in the drift chamber and the five-layer silicon detector.

To detect photons and electrons, both B-factory detectors use large arrays of CsI(Tl) crystals located inside the coil of the magnet. In BABAR and Belle, another novel feature is the use of resistive plate chambers (RPC) inserted into the steel return yoke of the magnet. This detector system is used for both muon and K_L detection and extends the measurement to the \( B_0 \rightarrow J/\psi K^0_L \) decay modes in addition to the \( CP \) eigenstate \( B \rightarrow J/\psi K^0_{SL} \).

To read out the detectors, BABAR uses electronics based on digital pipelines and incurs little or no dead-time. Belle uses charge-to-time (Q-to-T) convertors that are then read out by multihit time-to-digital counters (TDCs). This allows a uniform treatment of timing and charge signals.

3. RECONSTRUCTION OF B MESONS

The \( b \rightarrow c\bar{c}s \) decays to \( CP \) eigenstates used for the determination of \( CP \)-violation parameters are \( J/\psi K^0_{SL}, \psi(2S)K^0_{SL}, \chi_{c1} K^0_{SL}, \eta_c K^0_{SL}, J/\psi K^{*0}(K^{*0} \rightarrow K^{0}_S \pi^0), \) and \( J/\psi K^0_S \). In addition, in order to measure the performances of the tagging algorithm and check for biases, flavor eigenstates are reconstructed, with \( B \) mesons decaying either hadronically \( (B^0 \rightarrow D^{(*)-}\pi^+, D^{(*)-}\rho^+, D^{(*)-}\phi^+, J/\psi K^{*0}(K^{*0} \rightarrow K^{0}_S \pi^0)) \) or semileptonically \( (B^0 \rightarrow D^{(*)-}\ell^+\nu) \).

The reconstruction starts with the selection of multihadron events. Low-multiplicity events and beam-related backgrounds are suppressed by requiring that the event contain at least three charged tracks that are consistent with coming from the interaction point. Backgrounds from \( \gamma\gamma \) or residual beam gas interactions are removed by requirements on the visible energy and/or calorimetric energies, e.g., the total measured energy is required to be large. In order to suppress continuum background, which has a jet-like shape as opposed to the spherical shape of the \( B\bar{B} \) events, the normalized second Fox-Wolfram moment \( R_2 \), which is a measure of the sphericity of the event, is required to be small.

\( B \) candidates in all modes, except for \( B^0 \rightarrow J/\psi K^0_{SL} \) and \( D^{(*)-}\ell^+\nu \), are fully reconstructed from all the final-state charged and neutral decay products. Intermediate mesons (such as \( D^{(*)-}, J/\psi, \ldots \) ) are identified from combinations of tracks and neutral objects and used to construct the \( B \)-meson candidates. The effective mass has to fall in a mode-dependent interval. For intermediate mesons with small intrinsic width (e.g., \( J/\psi \) or \( D^{(*)} \)), a kinematic fit imposing a mass constraint
on the four-momenta of the candidate’s daughters is applied in order to improve resolution on kinematic quantities.

To exploit the fact that the center-of-mass four-momentum of the $\Upsilon(4S)$ decay is well known, a pair of nearly uncorrelated kinematic variables are defined to discriminate against combinatorial background: the difference, $\Delta E$, between the energy of the $B$ candidate and the beam energy in the $\Upsilon(4S)$ center-of-mass frame, and the beam-energy substituted mass, $m_{ES}$ (also known as beam-constrained mass or beam-energy constrained mass), defined as

$$m_{ES} = \sqrt{\frac{1}{4} s - \mathbf{p}^* \cdot \mathbf{p}};$$

where $s$ is the square of the center-of-mass energy and $\mathbf{p}^*$ is the $B$-candidate momentum in the center-of-mass frame.

Figure 2 shows the correlation between these two variables and their projections for a sample of $B^0 \to J/\psi K^0_S$ candidates. The signal distributions are Gaussian for both variables and centered at $\Delta E = 0$ and $m_{ES} = M_B$, whereas the combinatorial background does not peak in either of these variables. The $m_{ES}$ resolution (about 2.5 MeV/$c^2$) depends primarily on the beam-energy spread and is therefore independent of the $B$-decay mode. It is an order of magnitude better than the resolution in the invariant mass reconstructed from the decay products because the $B$ candidate is nearly at rest and the beam energies have a narrow spread. In contrast, the $\Delta E$ resolution is strongly dependent on the details of the reconstruction and the resolution of the detector. The signal region is therefore defined by $m_{ES} > 5.27$ GeV/$c^2$ and by a $\Delta E$ window that differs from mode to mode.

The combinatorial background $m_{ES}$ distribution is typical of a phase-space process with a kinematic threshold. A good description of it is provided by an empirical function introduced by ARGUS (22):

$$A(m_{ES}; m_0, \xi) = A_B m_{ES} \sqrt{1 - \left(\frac{m_{ES}}{m_0}\right)^2} e^{\xi \left(1 - \left(\frac{m_{ES}}{m_0}\right)^2\right)}.$$

The $\Delta E$ distribution is typically linear for combinatorial background, but it peaks at $\Delta E \sim \pm 140$ MeV/$c^2$ for events in which the final state differs from the signal by only one pion.

The decays $B^0 \to J/\psi K^0_L$ and $B^0 \to D^{*-} \ell^+ \nu$ require special treatment because the $B$-factory detectors cannot measure the $K^0_L$ energy and the neutrino goes undetected.

### 3.1. Reconstruction of Intermediate Meson States

The candidate $J/\psi$ and $\psi(2S)$ mesons are reconstructed using their decays to lepton pairs, i.e., $J/\psi \to \mu^+ \mu^-$ and $e^+ e^-$. The $\psi(2S)$ meson is also reconstructed via its $J/\psi \pi^+ \pi^-$ decay, the $\chi_c1$ meson via its $J/\psi \gamma$ decay, and the $\eta_c$ meson via its $K^+ K^- \pi^0$, $K_S^0(\pi^+ \pi^-)K^\pm \pi^\mp$, and $p \bar{p}$ decays.
Figure 2  (a) Scatter plot of $\Delta E$ versus $M_{bc}$ for $J/\psi K^0_S(\pi^+\pi^-)$ candidates in the Belle experiment. The box represents the signal region. (b) The $\Delta E$ projection with $5.270 < M_{bc} < 5.290$ GeV/$c^2$. (c) The $M_{bc}$ projection with $|\Delta E| < 0.04$ GeV. The enhancement below $\Delta E = -0.15$ GeV/$c^2$ is due to decay modes with additional pions such as $B \to J/\psi K^*$ decays.

$J/\psi$ and $\psi(2S) \to \ell^+\ell^-$ decays are reconstructed combining oppositely charged track pairs in which both tracks are identified as leptons. Lepton identification criteria depend on the decay mode and its level of purity. To account for final-state radiation and bremsstrahlung, the four-momenta of photon candidates in a narrow cone around the predicted direction of the radiated photons are added to the electron or positron candidate. The remaining radiative tail in the invariant mass distribution is accommodated by an asymmetric cut on the $J/\psi$ candidate mass (see Figure 3).

The final states $K^+\pi^-$, $K^+\pi^-\pi^0$, $K^+\pi^+\pi^-\pi^-$, and $K^0_S\pi^+\pi^-$ are used to reconstruct $\bar{D}^0$ candidates, whereas $D^-$ candidates are selected in the $K^+\pi^-\pi^-$ and $K^0_S\pi^-$ decay modes. At BABAR, only the dominant resonant mode $\bar{D}^0 \to K^+\rho^-, \rho^- \to \pi^-\pi^0$ is used to reconstruct $\bar{D}^0 \to K^+\pi^-\pi^0$. The charged tracks from the decay of the $D$-meson candidates are required to be consistent with coming from a common vertex.
Candidates for the decay $D^{*-} \rightarrow \bar{D}^{0}\pi^{-}$ are reconstructed by combining a $\bar{D}^{0}$ candidate with a pion. Because the pion is almost at rest in the decay $D^{*} \rightarrow D\pi$, the mass difference $\Delta M(D^{*} - D^{0})$ between the $D^{*}$ candidates and the $D^{0}$ candidate has a narrow peak and provides good discrimination against background.

$\rho^{-}$ candidates are reconstructed from $\pi^{-}\pi^{0}$ pairs, whereas the $K^{*0}$ are reconstructed in the $K^{+}\pi^{-}$ and $K_{S}^{0}\pi^{0}$ modes. Pairs of photon candidates, assumed to come from the origin, are used to reconstruct $\pi^{0}$ candidates. $K_{S}^{0}$ candidates are reconstructed from either a pair of charged tracks or a pair of $\pi^{0}$ candidates. In the first case, the two charged tracks are fitted to the same vertex, which improves the invariant mass resolution significantly, and the vertex is required to be displaced from the beam-interaction region. To select $K_{S}^{0} \rightarrow \pi^{0}\pi^{0}$ candidates, the decay vertex for which the invariant masses of two $\pi^{0}$ candidates are the most consistent with the nominal $\pi^{0}$ mass is chosen.

### 3.1.1. YIELDS AND BACKGROUNDS FOR FULLY RECONSTRUCTED B MESONS

Most of the background to these modes is combinatorial and can therefore be estimated from the $m_{ES}$ distribution (see Figure 4) by fitting the candidates in the $\Delta E$ signal region with an ARGUS function plus a Gaussian function representing the signal.
Figure 4  Example of a $m_{ES}$ distribution for fully reconstructed $B$-meson candidates in flavor eigenstates from the BABAR experiment. The result of a fit to the sum of an ARGUS function and a Gaussian function is superimposed.

Table 3 gives the yields and combinatorial backgrounds for the two experiments separately.

Backgrounds for which the final state is similar to the signal, except for one missing low-momentum particle (such as $B^+ \rightarrow J/\psi K^+ K^+$, $K^+ \rightarrow K^0 \pi^+$ as background to $B^0 \rightarrow J/\psi K^+$), have the same $m_{ES}$ distribution as the signal. They are therefore referred to as “peaking background” and are part of the signal yield. Because their $\Delta E$ distribution is significantly offset from the signal, the fraction of this background in the signal region is small (typically below 1%) and is accounted for in the final fit.

3.1.2. RECONSTRUCTION OF $B^0 \rightarrow J/\psi K^0$  The decay mode $B^0 \rightarrow J/\psi K^{*0}$, $K^{*0} \rightarrow K^0 \pi^0$ requires special treatment because of the presence of the $K^{*0}$ meson. This is an example of a decay into two vector mesons, and therefore it is not a $CP$ eigenstate. The fraction of $\eta_{CP} = -1$ is extracted from full angular analysis to $B \rightarrow J/\psi K^*$ decays, with the $K^*$ meson decaying into any possible mode, and is found to be $(16 \pm 4\%)$ by BABAR (23) and $(19 \pm 4\%)$ by Belle (24). The two $CP$ eigenstates can be separated on a statistical basis by exploiting their different distributions in the transversity angle, $\theta_{tr}$, the angle between the directions of $B^+$ from $J/\psi$ and the axis normal to the $K^{*0}$ decay plane in the $J/\psi$ rest frame.

Finally, because of the large width of the $K^{*0}$ meson, a significant fraction of inclusive $J/\psi$ background survives the selection. The largest sources of background are other $J/\psi K^*$ modes (“feed-across” background). The branching fraction and the time distributions of these modes are well known. More uncertain are the contributions from nonresonant $J/\psi K^0 \pi^0$ decays and from other inclusive $J/\psi$ decay modes, which are estimated by Monte Carlo simulation.
### TABLE 3
Number of events in the signal region \( (N_{ev}) \) and corresponding purity \( (\pi) \) for the two experiments

<table>
<thead>
<tr>
<th>Mode</th>
<th>BABAR</th>
<th></th>
<th>Belle</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>( J/\psi K_0^0 (\pi^+ \pi^-) )</td>
<td>1429</td>
<td>96</td>
<td>1116</td>
<td>96</td>
</tr>
<tr>
<td>Other ( (c\bar{c}) K_0^0 )</td>
<td>721</td>
<td>85</td>
<td>523</td>
<td>86</td>
</tr>
<tr>
<td>( J/\psi K^{*0} (K_0^0 \pi^0) )</td>
<td>283</td>
<td>73</td>
<td>89</td>
<td>84</td>
</tr>
<tr>
<td>Fully reco. flavor eig.</td>
<td>32700</td>
<td>83</td>
<td>18045</td>
<td>82</td>
</tr>
<tr>
<td>( D^* \ell \nu )</td>
<td>30823*</td>
<td>78</td>
<td>47317</td>
<td>79</td>
</tr>
</tbody>
</table>

*Only the first 30 fb\(^{-1}\) are used as cross-check.*

### 3.2. Reconstruction of \( B^0 \rightarrow J/\psi K^0_L \)

The decay of \( B^0 \rightarrow J/\psi K^0_L \) is challenging to reconstruct because only the direction of the \( K^0_L \) is known. Although characterized by a lower detection efficiency and higher backgrounds than other fully reconstructed \( CP \) eigenstates, this mode is important because it allows the measurement of \( \sin 2\beta \) in a \( CP = +1 \) eigenstate.

The lifetime of the \( K^0_L \) mesons is so long that they usually interact in the electromagnetic calorimeter or the muon detector before they decay. Therefore, they can be identified as either calorimetric clusters (“calorimeter candidates”) or as showers in the muon detector (“neutral hadron candidates”). Calorimetric candidates can be separated from the primary background, constituted by electromagnetic showers from photons, by exploiting the fact that the \( K^0_L \) clusters are broader than electromagnetic ones: Shower shape variables are defined to separate the signal from the background. The energy of the \( K^0_L \) meson cannot be measured, but the position of the impact point can be estimated from the cluster centroid. Neutral hadron candidates are, on the other hand, defined as long and short sets of hits in the muon detector. Background from charged pions, kaons, or muons can be rejected by requiring the hits to be inconsistent with coming from any track in the event. The position of the impact point is estimated from the centroid of the hits. However, the resolution on the direction of flight is worse than in the case of the calorimetric candidates.

The two kinds of candidates are considered separately because their background levels are different. If a calorimeter candidate is close to a neutral-hadron candidate, the event is assigned to the neutral-hadron category (which is the cleanest), but the direction of flight is estimated from the electromagnetic calorimeter, since it is more accurate.

The laboratory momentum of the \( K^0_L \) is determined from its flight direction and the constraint that the invariant mass of the \( J/\psi K^0_L \) system is consistent with the known \( B^0 \) mass. Possible discriminators between signal and combinatorial...
background are the difference, $\Delta E_{KL}$, between the energy of the $J/\psi K^0_L$ system and the beam energy (used by BABAR) or the $J/\psi K^0_L$ momentum, $p_{B}^{\text{rms}}$ (used by Belle) (25), in the $\Upsilon(4S)$ frames. The two variables are not independent and only one of them is used. Figure 5 shows the $\Delta E_{KL}$ and $p_{B}^{\text{rms}}$ distributions of selected $B^0 \rightarrow J/\psi K^0_L$ combinations for the two categories of $K^0_L$ candidates.

3.2.1. YIELDS AND BACKGROUNDS FOR $B^0 \rightarrow J/\psi K^0_L$ CANDIDATES Because of tight requirements on the identification of the leptons from $J/\psi$ decays and the high purity of the $K^0_L$ selection, background in $B^0 \rightarrow J/\psi K^0_L$ is mostly due to other $B \rightarrow J/\psi K^0_L X$ decays. The components with fake $K^0_L$ or $J/\psi$ candidates are considered separately. The fraction of each of these components is extracted from a fit to the distribution of the discriminating variable ($\Delta E_{KL}$ or $p_{B}^{\text{rms}}$). The shapes of the different components are taken either from simulation or, in the case of the fake $J/\psi$ component, from $J/\psi$ mass sidebands or electron-muon pairs. The fit also determines the fraction of signal events. Table 4 gives the observed yield and total fitted background for the calorimeter and neutral hadron candidates.

Because some components of the background are also $CP$ eigenstates (e.g., $B^0 \rightarrow J/\psi K^{*0}, K^{*0} \rightarrow K^0_L \pi^0$), knowledge of the composition of the inclusive $J/\psi$ background is crucial. However, there are large uncertainties on the branching ratios and $CP$ content of the individual modes involved. These uncertainties are included in the final systematic error.

3.3. Reconstruction of $B^0 \rightarrow D^{*-} \ell^+ \nu$

Semileptonic decays of the $B$ mesons are more difficult to select than the fully reconstructed hadronic decay modes because the neutrino is undetected. However,
TABLE 4  Number of $B^0 \to J/\psi K^0_S$ signal ($N_S$) and background ($N_B$) calorimeter and neutral-hadron candidates

<table>
<thead>
<tr>
<th></th>
<th>Calorimeter</th>
<th>Neutral Hadron</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$N_S$</td>
<td>$N_B$</td>
</tr>
<tr>
<td>BABAR</td>
<td>433</td>
<td>491</td>
</tr>
<tr>
<td>Belle</td>
<td>542</td>
<td>294</td>
</tr>
</tbody>
</table>

these modes have large yields and are very useful to test the performances of the algorithms used in the analysis. BABAR uses these decays only as a control sample, whereas Belle also uses them for the extraction of the tagging dilutions.

Leptons selected with tight criteria are combined with energetic $D^*$ candidates of opposite charge. In order to reject backgrounds, BABAR and Belle exploit the facts that the $D^*$ and lepton candidates are more likely to be back-to-back and that the missing mass should be zero. BABAR requests that the cosine of the angle between the $D^*$ and the lepton candidate be less than zero and that the cosine of the angle between the $B$ and the $D^*\ell$ system, assuming zero missing mass, $(\cos \theta_{B-D^*\ell}) = \frac{2(p_B \cdot p_{D^*\ell} - m_B^2 - m_{D^*}^2)}{2|p_B||p_{D^*\ell}|} < -1.1$ and $+1.1$. In addition, Belle requires $C = 2|p_B||p_{D^*\ell}| \leq \frac{1}{1+0.16\cos \theta_{B-D^*\ell}^*}$.

The background to this mode is subdivided into four categories: continuum, fake $D^*$ candidates, true $D^*$ and lepton candidates not originating from the same meson, and other semileptonic decays ($D^*\pi\ell\nu$). The fraction of the first three background components is extracted from data control samples, whereas the fourth is obtained from a fit to either $\cos \theta_{B-D^*\ell}$ or $C (C = \frac{1}{2}[(E_B - E_{D^*\ell})^2 - |p_B^2 - p_{D^*\ell}^2|]/2|p_B|p_{D^*\ell}|)$, where there is good discrimination from the signal. Table 3 shows the number of signal candidates and the purity of the sample.

4. MEASUREMENT OF $\Delta t$

Time-dependent measurements require the reconstruction of the time difference between the two $B$-meson decays. This is feasible because of the boost of the $\Upsilon(4S)$, $\beta \gamma \sim 0.56(0.43)$ in BABAR (Belle), and the silicon detectors that provide accurate measurements of the trajectories of charged particles.

The time difference $\Delta t$ can be approximately related to the spatial separation $\Delta z$ between the decay points of the two $B$ mesons,

$$\Delta z = \beta \gamma \gamma_{reco}^* c \Delta t + \gamma \beta_{reco}^* \gamma_{reco}^* \cos \theta_{reco}^* c (\tau_B + |\Delta t|),$$

where $\theta_{reco}^*$ is the polar angle with respect to the beam direction, $\beta_{reco}^*$ the velocity, and $\gamma_{reco}^*$ the boost factor of the reconstructed $B$ in the $\Upsilon(4S)$ frame.
The “boost approximation,” used by Belle, drops all terms but the first one:

\[ \Delta t = \frac{\Delta z}{c \beta \gamma \gamma^*_{\text{reco}}} \]

The determination of \( \Delta z \) requires the measurement of the position of the vertices of both the fully reconstructed \( B \) candidate (\( B_{\text{reco}}, e.g., B^0 \rightarrow J/\psi K^0_S \)) and the other \( B \), which is usually not completely reconstructed (\( B_{\text{tag}} \)). The reconstruction of the \( B_{\text{reco}} \) vertex is affected only by the resolution on the trajectory of the individual tracks that constitute the candidate CP eigenstate. The “tag” vertex reconstruction is impacted by the additional uncertainty regarding the origin of the tracks. For instance, if tracks from charmed mesons are included in the vertex determination and assigned to the primary vertex, they can introduce a bias. By definition, \( \Delta z = z_{\text{reco}} - z_{\text{tag}} \); thus, charmed meson decays induce a negative bias in \( \Delta t \).

4.1. \( \Delta t \) Reconstruction

The \( B_{\text{reco}} \) vertex is reconstructed from charged tracks and photon candidates that are combined to make up intermediate mesons (e.g., \( J/\psi, D, K_S \)) and then treated as virtual particles. The trajectory of these virtual particles is computed from those of their decay particles, and, when appropriate, mass constraints are imposed to improve the knowledge of the kinematics. In the case of charmonium states such as \( J/\psi K^0_S \), Belle uses only the dileptons from the \( J/\psi \) decay. In Belle, the vertex of the signal candidate is constrained to come from the beam-spot in the \( x - y \) plane and convolved with the finite \( B \)-meson lifetime. \( \text{BABAR} \) uses the beam-spot information only in the tag vertex reconstruction. The resulting spatial resolution depends on the final state; it is typically \( \sim 65 \mu m \) in \( \text{BABAR} \) and \( \sim 75 \mu m \) in Belle.

\( \text{BABAR} \) determines the \( B_{\text{tag}} \) vertex by exploiting the knowledge of the center-of-mass four-momentum and an estimate of the interaction point or beam-spot position. This information, along with the measured three-momentum of the fully reconstructed \( B_{\text{reco}} \) candidate, its decay vertex, and its error matrix, permits calculation of the \( B_{\text{tag}} \) production point and three-momentum, with its associated error matrix (see Figure 6). All tracks that are not associated with the \( B_{\text{reco}} \) reconstruction are considered; \( K^0_L \) and \( \Lambda^0 \) candidates are used as input to the fit in place of their daughters, but tracks consistent with photon conversions (\( \gamma \rightarrow e^+ e^- \)) are excluded. To reduce the bias from charm decay products, the track with the largest vertex \( \chi^2 \) contribution greater than 6 is removed and the fit is iterated until no track fails the \( \chi^2 \) requirement.

\( \text{Belle} \) reconstructs the \( B_{\text{tag}} \) vertex from well-reconstructed tracks that have hits in the silicon vertex detector and are not assigned to the \( B_{\text{reco}} \) vertex. Tracks from \( K^0_L \) candidates and tracks farther than 1.8 mm in \( z \) or 500 \( \mu m \) in \( r - \phi \) from the \( B_{\text{reco}} \) vertex are excluded. An iterative fit to these tracks is performed with the constraint that the vertex position be consistent with the beam spot. If the overall \( \chi^2 \) is poor, the track with the worst \( \chi^2 \) contribution is removed, unless it is identified as a
Figure 6  Schematic view of the geometry in the $yz$ plane for a $\Upsilon(4S) \to B\bar{B}$ decay. For fully reconstructed decay modes, the line of flight of the $B_{\text{tag}}$ can be estimated from the (reverse) momentum vector and the vertex position of $B_{\text{reco}}$, and from the beam-spot position in the $xy$ plane and the $\Upsilon(4S)$ average boost. Note that the scale in the $y$ direction is substantially magnified compared to that in the $z$ direction.

high-momentum lepton. In this case, the lepton is retained and the track with the second-largest $\chi^2$ is removed.

The resolution on $\Delta z$ is dominated by the $B_{\text{tag}}$ vertex reconstruction and therefore is nearly independent of the reconstructed $CP$ decay mode. Based on Monte Carlo simulation, it is estimated to be $\sim 190 \mu m$. The $\Delta z$ measurement is converted to a $\Delta t$ measurement, and the corresponding resolution is 1.1 ps in $BABAR$ and 1.43 ps in Belle because of the different center-of-mass boosts.

4.2. $\Delta t$ Resolution Function

Because the resolution in $\Delta t$ is of the order of the $B$ lifetime (1.2 ps), knowledge of the resolution function is crucial. The total resolution function can be described as a sum of three Gaussians, representing the core, tail, and outlier components. The outlier component accounts for a small ($<0.1\%$) fraction of misreconstructed events and has large, fixed resolution and zero mean.

The means of the core and tail Gaussians account for the bias from charm decay. Because the $D$-meson flight direction is correlated with the resolution on $\Delta z$ calculated from the propagation of the covariance matrices, $\sigma_{\Delta z}$, the means are parameterized as linear combinations of this variable. In the $BABAR$ analysis, the coefficients of the linear combination depend on the tagging category and are left free in the likelihood fit to the data, whereas in the Belle analysis they are determined from data control samples (the constant terms) or Monte Carlo simulation (the linear coefficients).

The resolutions of the core and tail Gaussians ($\sigma_{\Delta z}^{\text{core}}$ and $\sigma_{\Delta t}^{\text{tail}}$) are also functions of $\sigma_{\Delta z}$, but the parameterization is different for the two experiments. $BABAR$
computes $\sigma_1$, propagating the errors according to Equation 9, and then assumes a linear scaling of the true errors ($\sigma_{\text{core}} = S_{\text{core}} \sigma_1$ and $\sigma_{\text{tail}} = S_{\text{tail}} \sigma_1$). $S_{\text{core}}$ is a free parameter in the likelihood fit and is measured on data to be $1.094 \pm 0.048$. $S_{\text{tail}}$ is instead fixed to 3 and varied in the evaluation of the systematic error, but the fraction of events in the second Gaussian is floated in the fit.

Belle computes the $\Delta t$ resolutions from the convolution of four contributions: the detector resolution on $\Delta z$ for the $B_{\text{reco}}$ and $B_{\text{tag}}$ vertices, the broadening due to the inclusion of tracks from charm and $K_0^0$ decays in the tag vertex, and the uncertainty due to the boost approximation (26). The uncertainties due to the boost approximation and the impact of nonprompt tracks are estimated from Monte Carlo simulation. The intrinsic $\Delta z$ resolutions are obtained from $\sigma_{\Delta z}$ applying corrections that account for underestimated errors on the single-track trajectory parameters and for the effects of the charm decays. The former set of corrections are extracted from data because they depend on the detector performance, whereas the latter are determined from Monte Carlo simulation. The majority of vertices include several tracks. However, about 10% of $B_{\text{reco}}$ vertices and 22% of tag vertices are obtained from only a single track and the beam-spot constraint. The Belle resolution function treats these two classes of vertices separately.

The parameters of the resolution function are extracted from data and are used for measurements of $B$ lifetimes (27, 28) and the $B-\bar{B}$ mixing frequency (29–32). Good agreement between data and Monte Carlo simulation is found even in the tails of the time distributions. This is illustrated in the $B^0$ and $B^+$ lifetime distributions shown in Figure 7.

5. DETERMINATION OF THE $b$ FLAVOR OF THE ACCOMPANYING B MESON

After the decay products of the $B$ decaying to a $CP$ eigenstate ($B_{\text{reco}}$) have been identified, all of the remaining particles belong to the decay of the other $B$ meson ($B_{\text{tag}}$). To observe time-dependent $CP$ asymmetries, the $b$-quark flavor of the second $B$ has to be identified (tagged).

The simplest and most reliable method for flavor tagging uses the charge of high-momentum leptons in semileptonic decays, i.e., $B^0 \rightarrow X \ell^+\nu$ and $\bar{B}^0 \rightarrow X \ell^-\bar{\nu}$. The charges of final-state kaons can also be used as tags because the inclusive processes $B^0 \rightarrow K^+X$ ($\bar{b} \rightarrow \bar{c} \rightarrow \bar{s}$) and $\bar{B}^0 \rightarrow K^-X$ ($b \rightarrow c \rightarrow s$) dominate. Furthermore, a significant fraction of neutral $B$ decays contains $D^{\pm}$ mesons produced via $B^0 \rightarrow D^{\pm}X^\mp$ or $\bar{B}^0 \rightarrow D^{\mp}X^\pm$. The decay $D^{\pm} \rightarrow D^{0}\pi^\pm$ has a large branching fraction, and the charge of the low-momentum pion (“soft pion”) produced in this decay can also be used to identify the $B$ flavor. The soft pion momentum vector tends to follow the $D^*$ direction and can be distinguished from background by the angular correlation with the thrust axis of the tagged $B$.

These discriminating features provide most of the tagging separation and are used by both experiments. In addition, Belle’s tagging algorithm includes other categories of tracks with charges that depend on the $b$ quark’s flavor:
lower-momentum leptons from $c \to s \ell^+ \nu$; $\Lambda$ baryons from the cascade decay $b \to c \to s$; and high-momentum pions that originate from quasi-two-body decays such as $B^0 \to D^{(*)-} \pi^+, \rho^+, a_1^+$. Both experiments combine all the tagging information, taking correlations between variables into account, in a way that maximizes the flavor-tagging performance. The performance is characterized by two parameters: $\epsilon$ is the raw tagging efficiency, and $w$ is the probability that the flavor tag is wrong (wrong-tag fraction).

A nonzero value of $w$ results in a dilution of the true asymmetry. For example, if the true numbers of reconstructed $B^0$ and $\bar{B}^0$ are $n_{B^0}$ and $n_{\bar{B}^0}$, the corresponding asymmetry is $A = (n_{B^0} - n_{\bar{B}^0})/(n_{B^0} + n_{\bar{B}^0})$. Taking wrong tags into account, the observed numbers are $N_{B^0} = \epsilon((1 - w)n_{B^0} + wn_{\bar{B}^0})$ for $B^0$ and $N_{\bar{B}^0} = \epsilon((1 - w)n_{\bar{B}^0} + wn_{B^0})$ for $\bar{B}^0$, and the observed asymmetry becomes $(1 - 2w)A$. The statistical error of the measured asymmetry is proportional to $\epsilon^{-1/2}$, and the number of events required to observe the asymmetry for a certain statistical significance is inversely proportional to $\epsilon_{\text{eff}} = \epsilon(1 - 2w)^2$, which is called the effective efficiency.

An imperfect knowledge of $w$ can be a significant source of systematic error, and therefore it is necessary to extract $w$ from data.
5.1. Flavor-Tagging Method

Flavor tagging is achieved by means of multivariate techniques that are divided into a track stage and an event stage.

At the track stage, individual tracks are assigned to one or more of the following categories: lepton-like, kaon-like, and soft-pion-like. Belle also has a \( \Lambda \) category and subdivides its kaon category into two parts, one for events with \( K_S^0 \) decays and the other for events without \( K_S^0 \). Separate treatment is necessary because events with \( K_S^0 \) have a larger wrong-tag fraction owing to their additional strange-quark content.

For each track, several discriminant variables are computed. For instance, in the case of leptons, variables include the charge, the likelihood of being an electron or a muon as calculated from the output of the particle-identification algorithms, the momentum in the \( \Upsilon(4S) \) rest frame, the missing momentum of the event, and its direction with respect to the lepton candidate. These variables are then combined with a multivariate algorithm to return a discrete and a continuous variable. The discrete variable, \( q \), represents the \( b \) flavor and is \( q = +1 \) (\( q = -1 \)) if the event is more likely to be a \( B^0 (\bar{B}^0) \). The continuous one, \( r \), is an estimate of the probability that the \( b \) flavor is correctly tagged and therefore ranges from \( r = 0 \) to \( r = 1 \).

\textit{BABAR} uses neural networks trained on simulated samples. The output \( z \) of each neural network ranges between \( z = -1 \) (if an event is certainly \( \bar{B}^0 \)) and \( z = +1 \) (certainly \( B^0 \)). Belle instead uses a likelihood-based method. This is implemented with a look-up table, which, as a function of the input variables, returns \( z = \frac{N_B}{N_B + N_{\bar{B}}} \) as estimated on simulated events. In both cases, one can define \( q \) as the sign of \( z \) and \( r \) as its absolute value.

The outputs of the kaon and \( \Lambda \) categories are combined to obtain a global \( q \) and \( r \), and the best candidates among the outputs of the lepton and soft-pion categories are chosen. These quantities are used as inputs to the event stage and are combined with a multivariate algorithm of the same kind as that used for the track stage. The resulting value of \( q \) is adopted to label an event as \( B^0 (q = +1) \) or \( \bar{B}^0 (q = -1) \).

Events are then divided into several mutually exclusive categories with different probabilities of being correctly tagged. \textit{BABAR} uses a hierarchy of categories identified by the physics content of the event. If an event has a good value at the lepton track stage \( r \), and all kaons give an answer consistent with the lepton, it is assigned to the lepton category. If both a kaon and a soft pion with the same \( q \) are present, or if there are one or more consistent kaons with high \( r \), the event is assigned to the Kaon I category. All other events with one or more kaons are assigned to the Kaon II category. The remaining events with high \( r \) are assigned to the inclusive category. Belle subdivides the events according to the overall \( r \) value. Six bins are separated by \( r = 0, 0.25, 0.5, 0.625, 0.75, 0.875, 1.0 \).

5.2. Flavor-Tagging Performances

Although the algorithm for the calculation of \( r \) is defined using simulated events, the probability of obtaining a correct tag and the mistag probability \( w \) in each category are evaluated directly from data.
The flavor-tagging performance is estimated with fully reconstructed flavor eigenstates such as $B^0 \rightarrow D^{(*)-}\pi^+$, $D^{(*)-}\rho^+$, $D^{(*)-}a_1^+$, and $J/\psi K^{*0}(K^+\pi^-)$. Belle also adds semileptonic decays, $B^0 \rightarrow D^{*-}l^+\gamma$, to this sample, whereas BABAR uses these decays only as a control sample.

The flavors of both $B$ mesons are known, and the time evolution of neutral $B$-meson pairs with opposite flavor (OF) or same flavor (SF) are given by

$$\mathcal{P}_{\text{OF}}(\Delta t) \propto 1 + (1 - 2w) \cos(\Delta m_d \Delta t),$$

$$\mathcal{P}_{\text{SF}}(\Delta t) \propto 1 - (1 - 2w) \cos(\Delta m_d \Delta t).$$

The $(CP$-conserving) OF-SF asymmetry exhibits a $\cos(\Delta m_d \Delta t)$ modulation characteristic of $B^0 - \bar{B}^0$ mixing and is given by

$$A_{\text{mix}} \equiv \frac{\mathcal{P}_{\text{OF}} - \mathcal{P}_{\text{SF}}}{\mathcal{P}_{\text{OF}} + \mathcal{P}_{\text{SF}}} = (1 - 2w) \cos(\Delta m_d \Delta t),$$

where $w$ is the wrong-tag fraction. The amplitude of $A_{\text{mix}}$ is proportional to $(1 - 2w)$, and its frequency depends on $\Delta m_d$.

It is thus possible to obtain the value of $w$ directly from the data by fitting the $\Delta t$ distribution of the SF and OF events with $\Delta m_d = 0.472 \text{ ps}^{-1}$ fixed at the world-average value (35). The small uncertainty in this parameter is considered in the systematic-error evaluation. The procedure used to form the probability density function for the fit is similar to that adopted for the maximum-likelihood analysis of $CP$ eigenstates, which is described in Section 6.

Figure 8 shows the measured OF-SF asymmetries as a function of $\Delta t$ for $B \rightarrow D^{*+}l^+\nu$ events tagged by Belle in the six $r$ categories. The curves in the figure are obtained by the fit; the background is not subtracted. As the $r$ value increases and the probability of correct $B$-flavor determination improves, the decay-time-dependent $B^0 - \bar{B}^0$ mixing signal becomes clearer.

For hadronic modes, the asymmetries $A_{\text{mix}}$ are similar to those for semileptonic $B$ decays. BABAR performs a simultaneous fit to mixing and $\sin 2\beta$, in order to take into account correlations among the parameters (see Section 6). Table 5 summarizes the resulting efficiency, effective efficiency, and mistag rates as extracted from the fit to the hadronic and semileptonic events for Belle, and from the global fit for BABAR. The two experiments have the same tagging performances within errors ($\epsilon_{\text{eff}} \sim 28.5\%$), although BABAR restricts the measurement to 66% of the events, whereas Belle considers all of them.

### 6. LIKELIHOOD FIT

The value of $\sin 2\beta$ is extracted using an unbinned maximum-likelihood technique based on the $f_\pm$ functions in Equation 4 ($\pm$ refers to the $b$ flavor, determined by the tagging algorithm described in Section 5.1). In order to account for vertexing resolution and for incorrect tagging, the $f_\pm$ functions are convolved with the
Figure 8  Measured asymmetry $A_{\text{mix}}$ (Equation 12) as a function of the decay time difference $\Delta t$ for six regions of $r$ obtained for the $B^0 \rightarrow D^{\ast} \ell \nu$ control sample in Belle. The background is not subtracted. The results of the fit are shown as curves.

resolution function ($R$, described in Section 4.2) and the dilutions $D_c$ per tagging category ($c$) are introduced (see Section 5):

$$P_{\text{sig,}\Delta t}(\Delta t, c) = [D_c f_{\Delta t}(\Delta t') + (1 - D_c) f_{\Delta t'}(\Delta t')] \otimes R(\Delta t - \Delta t').$$  

$BABAR$ allows for a possible dependence of the tagging dilutions on the $b$ flavor, recognizing that the probability of misidentifying a $B^0$ is different from
the probability of misidentifying a $\bar{B}^0$, whereas Belle includes this effect in the systematic error. In the case of the $B^0 \to J/\psi K^{*0}$ decays, Belle separates the $CP$-odd and $CP$-even components by means of the transversity angle $\theta_t$ (see Section 3.1.2).

Both the resolution function and the dilutions are common to the $CP$ and the flavor eigenstate samples. The underlying likelihood is different because in the latter case only the $\cos(\Delta m_d \Delta t)$ term due to $B \bar{B}$ mixing is present. Therefore, it is possible to estimate all the fit parameters other than $\sin 2\beta$ from the flavor eigenstate sample, either by assuming the mixing frequency or by determining it as a free parameter in the fit. Belle adopts a two-step procedure, first extracting the parameters from control samples and then using them in the $\sin 2\beta$ fit, whereas BABAR performs a simultaneous fit to flavor and $CP$ eigenstates and allows all parameters to float. The correlations among parameters are easier to take into account in this way, although the fit is more complicated. Including the background parameters discussed below, the fit has 34 free parameters, but the largest correlation of $\sin 2\beta$ with any linear combination of them is only 13%.

### 6.1. Background Parameterization

Backgrounds are taken into account by including a decay-mode-dependent term in the probability density function:

### TABLE 5  Tagging efficiency ($\epsilon$), mistag rate ($w$), and effective efficiency ($\epsilon_{\text{eff}}$) for each tagging category

<table>
<thead>
<tr>
<th>Category</th>
<th>$\epsilon$</th>
<th>$w$</th>
<th>$\epsilon_{\text{eff}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>BABAR</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lepton</td>
<td>0.091</td>
<td>0.033 ± 0.006</td>
<td>0.079 ± 0.003</td>
</tr>
<tr>
<td>Kaon I</td>
<td>0.167</td>
<td>0.100 ± 0.007</td>
<td>0.107 ± 0.004</td>
</tr>
<tr>
<td>Kaon II</td>
<td>0.198</td>
<td>0.209 ± 0.008</td>
<td>0.067 ± 0.004</td>
</tr>
<tr>
<td>Inclusive</td>
<td>0.200</td>
<td>0.315 ± 0.009</td>
<td>0.027 ± 0.003</td>
</tr>
<tr>
<td>Total</td>
<td>0.656</td>
<td></td>
<td>0.281 ± 0.007</td>
</tr>
<tr>
<td>Belle</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$r &gt; 0.875$</td>
<td>0.136</td>
<td>0.020 ± 0.006</td>
<td>0.126 ± 0.003</td>
</tr>
<tr>
<td>$0.75 &lt; r &lt; 0.875$</td>
<td>0.094</td>
<td>0.112 ± 0.009</td>
<td>0.056 ± 0.003</td>
</tr>
<tr>
<td>$0.625 &lt; r &lt; 0.75$</td>
<td>0.122</td>
<td>0.160 ± 0.009</td>
<td>0.056 ± 0.003</td>
</tr>
<tr>
<td>$0.5 &lt; r &lt; 0.625$</td>
<td>0.104</td>
<td>0.228 ± 0.010</td>
<td>0.031 ± 0.002</td>
</tr>
<tr>
<td>$0.25 &lt; r &lt; 0.5$</td>
<td>0.146</td>
<td>0.336 ± 0.009</td>
<td>0.016 ± 0.002</td>
</tr>
<tr>
<td>$r &lt; 0.25$</td>
<td>0.398</td>
<td>0.458 ± 0.006</td>
<td>0.003 ± 0.001</td>
</tr>
<tr>
<td>Total</td>
<td>1.0</td>
<td>0.288 ± 0.006</td>
<td></td>
</tr>
</tbody>
</table>
\[ P_{\pm}(\Delta t, c) = f_{\text{sig}} P_{\text{sig}, \pm}(\Delta t, c) + (1 - f_{\text{sig}}) P_{\text{bkg}, \pm}(\Delta t, c). \]

The signal fractions, \( f_{\text{sig}} \), are functions of \( m_{\text{ES}} \) in the case of fully reconstructed modes: A sum of an ARGUS function empirical background and a Gaussian for the signal is adopted (see Figure 4). In addition, a small (∼1%) component with the same \( m_{\text{ES}} \) distribution as the signal but a different \( \Delta t \) distribution is included in the background in order to account for the peaking background (see Section 3.1.1). In the case of the \( B^0 \to J/\psi K^0_L \) mode, the signal fractions are a function of the discriminating variable \( \Delta E_{KL} \) for BABAR and \( p_{B}^{\text{cms}} \) for Belle and are extracted from the likelihood fit described in Section 3.2.1. In the case of the \( B^0 \to J/\psi K^{*0} \) mode, the signal fraction is estimated directly from simulation (see Section 3.1.2).

The \( P_{\text{bkg}, \pm} \) distributions for the combinatorial background in the fully reconstructed \( B \) samples are empirically parameterized as the sum of a component with an effective lifetime and a prompt component. The relative fraction of the two components is determined from the \( m_{\text{ES}} \) and \( \Delta E \) sidebands in data. For \( B^0 \to J/\psi K^0_L \), the background distribution from \( B \to J/\psi X \) decays is the sum of a number of exclusive components taken from Monte Carlo simulation. The fractions of each component and their effective \( CP \) content are varied in the systematic error. The non-\( J/\psi \) distribution is extracted from data sidebands and its fraction taken from the likelihood fit to the final discriminating variable. Finally, the \( P_{\text{bkg}, \pm} \) distributions for \( B^0 \to J/\psi K^{*0} \) are parameterized as the sum of three terms, from feed-across, nonresonant, and combinatorial background, discussed in Section 3.1.2. Both the relative fractions and the individual time dependences are extracted from simulation and varied in estimating the systematic error.

### 6.2. Direct \( CP \) Violation

While the likelihood fits used to extract the primary results are performed with the standard-model expectation \(|\lambda| = 1\), a search for the effect of direct \( CP \) violation is also made, utilizing the full expression of Equation 4 and fitting for \(|\lambda|\) and for the coefficient of the \( \sin(\Delta m \Delta t) \) term, \( A \), separately.

A significant part of the sensitivity to \(|\lambda|\) comes from the difference in \( B^0 \) and \( \bar{B}^0 \) yields. Possible differences in tagging efficiencies are therefore checked in the flavor eigenstate sample and taken into account in the fit.

### 7. RESULTS

Table 6 summarizes the fit results for individual decay modes for Belle and BABAR separately. All decay modes give consistent results.

The value of \( \sin 2\beta \) measured by BABAR is (16)

\[ \sin 2\beta_{\text{BABAR}} = 0.741 \pm 0.067 \text{(stat.)} \]
TABLE 6  \[ \sin 2\beta \] fit results for BABAR and Belle separately, for the entire \( CP \) sample, and for subsamples

<table>
<thead>
<tr>
<th>Mode</th>
<th>BABAR</th>
<th>Belle</th>
</tr>
</thead>
<tbody>
<tr>
<td>( J/\psi K_S^0(\pi^+\pi^-) )</td>
<td>0.82 ± 0.08</td>
<td>0.73 ± 0.10</td>
</tr>
<tr>
<td>All ( \eta_f = -1 )</td>
<td>0.76 ± 0.07</td>
<td>0.72 ± 0.08</td>
</tr>
<tr>
<td>( J/\psi K_L^0 )</td>
<td>0.72 ± 0.16</td>
<td>0.78 ± 0.17</td>
</tr>
<tr>
<td>( J/\psi K^0 (K_S^0 \pi^0) )</td>
<td>0.22 ± 0.52</td>
<td>0.04 ± 0.63</td>
</tr>
<tr>
<td>All</td>
<td>0.74 ± 0.07</td>
<td>0.72 ± 0.07</td>
</tr>
</tbody>
</table>

and that measured by Belle is (17)

\[
\sin 2\beta_{\text{Belle}} = 0.719 \pm 0.074(\text{stat.}) \text{.} 
\]

Because the likelihood functions are parabolic in this parameter, a combined value can be obtained by weighting the individual results with their statistical errors:

\[
\sin 2\beta = 0.731 \pm 0.050(\text{stat.}) \text{.} 
\]

Figures 9 and 10 show the fit results superimposed on the \( \Delta t \) distributions of the events tagged as \( B^0 \) or \( \bar{B}^0 \) for the two experiments. The characteristic signature of \( CP \) violation is visible as an asymmetric tail toward positive (negative) values of \( \Delta t \) for \( B^0 \) tags and \( CP \) eigenstates with eigenvalue \( \eta_f = -1 \) or \( \bar{B}^0 \) tags with \( \eta_f = +1 \) (\( B^0 \) tags with \( \eta_f = -1 \) or \( \bar{B}^0 \) tags with \( \eta_f = +1 \)).

Figures 9 and 11 show the raw asymmetry, defined as

\[
A_{\text{raw}} = \frac{N_{B^0} - N_{\bar{B}^0}}{N_{B^0} + N_{\bar{B}^0}}, 
\]

as a function of \( \Delta t \), in the two experiments separately, for the decay modes with \( \eta_f = +1 \) or \( -1 \). The projection of the likelihood fit onto this variable is overlaid. In the absence of detector effects, this quantity should have a sinusoidal oscillation with the frequency of \( B-\bar{B} \) mixing and amplitude equal to \( \sin 2\beta \). Tagging and background dilutions reduce the amplitude of the oscillation. The effect of background dilution is visible in the asymmetry distribution for the higher background modes (i.e., \( B^0 \to \psi K_L^0 \)) in the \( \eta_f = +1 \) samples.

A fit allowing for direct \( CP \) violation (see Section 6.2) has also been performed. Using the clean \( \eta_{CP} = -1 \) sample, where no assumptions about the behavior of the background are required, BABAR measures

\[
|\lambda| = 0.948 \pm 0.051(\text{stat.}) \pm 0.030(\text{sys.}) \text{.} 
\]

whereas on the full \( CP \) sample, Belle measures

\[
|\lambda| = 0.950 \pm 0.049(\text{stat.}) \pm 0.025(\text{sys.}) \text{.} 
\]

Restricting the Belle sample to the same decay modes as BABAR, Belle measures

\[
|\lambda| = 0.96 \pm 0.06(\text{stat.}) \text{.} 
\]
systematic errors are given in Reference 33.) As pointed out by Nir (36), existing limits on charge asymmetry in semileptonic $B$ decay combined with the limit on direct $CP$ violation in $B^\pm \to J/\psi K^\pm$ decay give an even more restrictive constraint,

$$|\lambda| = 1.007 \pm 0.026.$$  

These results are consistent with standard-model expectations. The fitted values of the coefficient of the $\sin(\Delta m\Delta t)$ term, $A$, are also consistent with the measured values of $\sin 2\beta$ from the default fit.
7.1. Consistency Checks

As a check of consistency among subsamples, Table 7 shows the fit result per tagging category and for $B^0$ and $\bar{B}^0$ tags separately. Control samples, in which no significant $CP$ violation is expected in the standard model, are also fitted to test the analysis procedure for possible biases. All subsamples are consistent among themselves and no bias is observed in the control samples.

In order to verify that the likelihood fit returns an unbiased measurement and the correct statistical error, thousands of fast parameterized Monte Carlo simulations of single experiments were performed. The distribution of the results is consistent with a normal distribution, with no bias and the correct width. Fully simulated events are then used to test that the detector effects are correctly accounted for. In this case, $BABAR$ observes a bias of $0.014 \pm 0.005$ on $\sin 2\beta$ and corrects for it.

7.2. Systematic Error

Table 8 lists the contributions to the systematic error on $\sin 2\beta$. The dominant systematic error comes from the understanding of the vertexing resolution function. In order to estimate the validity of the adopted parameterization, which is purely phenomenological, different equally valid $\Delta t$ determinations are compared using several track selection criteria, several vertex quality criteria, and alternative vertexing algorithms. The spread of the results is adopted as the vertexing error. $BABAR$ also exploits Monte Carlo simulations of possible errors on the alignment between the silicon tracker and the drift chamber and adds the resulting error in quadrature to the corresponding variation in the result. Finally, all external parameters related to the measurement of $\Delta t$, the beam spot, the $Z$ scale, and the center-of-mass boost are varied.
Figure 11  Belle data: Raw asymmetry ($A_{raw}$) for (a) the entire $CP$ sample, where the sign of the asymmetry in $\eta_f \pm 1$ samples has been inverted; (b) $\eta_f = -1$; and (c) $\eta_f = +1$ samples separately. (d) $A_{raw}$ for a sample of flavor eigenstates for which no significant asymmetry is expected.

Belle extracts the uncertainty on the dilutions and the vertexing resolution function from control samples and then propagates them into the signal sample. For BABAR, because of its global-fit approach, these contributions are already included in the statistical error on the measurement. Nonetheless, the underlying assumption is that these parameters are the same for the flavor and the $CP$ eigenstates. This hypothesis is tested by Monte Carlo simulation and a systematic error is assigned to it. Finally, the underlying assumption that the vertexing resolution is the same for incorrect and correct tags is not valid. The impact of this incorrect assumption is estimated from simulation and included in the resolution-function systematic error.

Several input parameters to the measurement are varied within errors (35): The $B^0$ lifetime is varied in the range $1.548 \pm 0.032$ ps and the mixing frequency in the range $0.472 \pm 0.017$ ps$^{-1}$. 
TABLE 7  \( \sin 2\beta \) fit results by tagging category and for \( B^0 \) and \( \bar{B}^0 \) tags separately

<table>
<thead>
<tr>
<th>Mode</th>
<th>BABAR</th>
<th>Belle</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tagging category 1</td>
<td>0.79 ± 0.11</td>
<td>0.72 ± 0.09</td>
</tr>
<tr>
<td>Tagging category 2</td>
<td>0.78 ± 0.12</td>
<td>0.62 ± 0.15</td>
</tr>
<tr>
<td>Tagging category 3</td>
<td>0.73 ± 0.17</td>
<td>1.27 ± 0.36</td>
</tr>
<tr>
<td>( B^0 ) tags</td>
<td>0.76 ± 0.10</td>
<td>0.65 ± 0.12</td>
</tr>
<tr>
<td>( \bar{B}^0 ) tags</td>
<td>0.75 ± 0.10</td>
<td>0.77 ± 0.09</td>
</tr>
<tr>
<td>Fully reco. flavor eig.</td>
<td>0.02 ± 0.02</td>
<td>0.01 ± 0.02</td>
</tr>
</tbody>
</table>

*For Belle, tagging category 1 corresponds to the range \( 0.75 < r < 1 \), whereas categories 2 and 3 correspond to the ranges \( 0.5 < r < 0.75 \) and \( 0 < r < 0.5 \), respectively. For BABAR, category 1 refers to lepton tags; categories 2 and 3 are Kaon I and Kaon II tags.

The parameters of the \( m_{ES} \) distribution used to distinguish between signal and background are varied within errors. The peaking background fraction (see Section 3.1.1) is varied within the errors determined by simulation, and its \( CP \) content is varied conservatively over the maximum range of \( ±1 \).

The signal fraction and background composition for \( B^0 \to J/\psi K^0_s \) are varied consistently with the results of the fits described in Section 3.2.1. The composition of the inclusive \( J/\psi \) background is varied according to the efficiencies predicted by simulations and the measured branching fractions.

A possible fit bias is checked using the full simulation (see Section 7.1) within a limit that is set by the statistics of the Monte Carlo simulation. This limit is reported as a systematic uncertainty. In the case of BABAR, a shift of 0.014 is applied with a systematic uncertainty equal to the fraction of this bias that is not

TABLE 8  Contributions to the systematic error in \( \sin 2\beta \)

<table>
<thead>
<tr>
<th>Source</th>
<th>BABAR</th>
<th>Belle</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vertexing</td>
<td>0.014</td>
<td>0.022</td>
</tr>
<tr>
<td>Dilutions</td>
<td>0.012</td>
<td>0.015</td>
</tr>
<tr>
<td>Resolution function</td>
<td>0.009</td>
<td>0.014</td>
</tr>
<tr>
<td>Physics</td>
<td>0.005</td>
<td>0.007</td>
</tr>
<tr>
<td>( J/\psi K^0_s ) background</td>
<td>0.015</td>
<td>0.010</td>
</tr>
<tr>
<td>Signal &amp; Background</td>
<td>0.018</td>
<td>0.006</td>
</tr>
<tr>
<td>Fit bias</td>
<td>0.013</td>
<td>0.011</td>
</tr>
<tr>
<td>Total</td>
<td>0.034</td>
<td>0.035</td>
</tr>
</tbody>
</table>
understood (0.010). B\(\bar{A}B\)AR has also studied the impact of possible CP-violating effects on the \(B_{\text{tag}}\) side, effects that arise from the interference between \(b \to \bar{u}cd\) and \(b \to c\bar{u}d\) amplitudes (34). The effect is not incorporated in the likelihood, but the estimated bias is included in the systematic error (0.008).

8. FUTURE PROSPECTS

The measurements of time-dependent asymmetries in charmonium decay modes show that there is large CP violation in the \(B\)-meson system. The combined result for \(\phi_1\) (a.k.a. \(\beta\)) from the \(B\)-factory experiments, \(\sin(2\phi_1) = 0.731 \pm 0.050\), is consistent with indirect constraints on the unitarity triangle, as shown in Figure 12. The time-dependent measurements of CP asymmetries now determine the angle \(\phi_1\) more precisely than the indirect determinations. Several possible solutions for \(\phi_1\) are allowed, but the solution in the upper left-hand portion of the \(\rho, \eta\) plane is theoretically preferred (36).

In these time-dependent CP-asymmetry measurements, the fit determines \(\sin(2\phi_1)\) rather than the angle \(\phi_1\). As a result, there are often multiple solutions or ambiguities for the angle \(\phi\) (e.g., four solutions for \(\phi_1\) from the initial \(\sin(2\phi_1)\) measurements). In the future, it should be possible to eliminate the multiple solutions for \(\phi_1\) with additional measurements. These measurements usually require high statistics and involve measurements of quantities that depend on other functions of the angles, such as \(\cos(2\phi_1)\) (38).

Within the next five years, there will also be precise measurements of \(\sin(2\phi_1)\) from hadron-collider experiments such as CDF and DØ. The Tevatron is expected to deliver 2 \(fb^{-1}\) of data. This will allow measurements with a statistical accuracy of \(\sim 0.03\). A comparable precision should also be attained at the \(B\) factories for data samples with integrated luminosities of 500 \(fb^{-1}\). After 2006, dedicated \(B\) experiments at hadron colliders, such as LHCB and BTeV, are planned and the possibility of building very-high-luminosity \(e^+e^-\) \(B\) factories is also being discussed (39).

Experimental work on the determination of the other angles, \(\phi_2\) and \(\phi_3\), has also started. To obtain precise results that test the validity of the standard model, much larger data samples will be required. We briefly describe some of the possible methods.

To measure \(\phi_2\) (a.k.a. \(\alpha\)), a promising approach involves the use of the decay to a CP eigenstate, \(B^0 \to \pi^-\pi^+\). The interference in this mode between the direct decay and the decay via mixing leads to a CP-violating asymmetry proportional to \(\sin 2\phi_2\) with the same mechanism as in the charmonium decay mode \(B^0 \to J/\psi K_S^0\).

However, there are several additional complications that are not present for decays involving charmonium states. The decay amplitude for \(B^0 \to \pi^+\pi^-\) contains a contribution from a tree diagram as well as a Cabibbo suppressed penguin diagram (40). The penguin contribution is not negligible and has a weak phase that
Figure 12  Indirect constraints on the position of the apex of the unitarity triangle in the ($\bar{\rho}$, $\bar{\eta}$) plane, including the direct measurement of $\sin(2\phi_1)$. The fitting procedure is described in Reference 37. The direct measurement $\sin(2\phi_1) = 0.731 \pm 0.050$ is represented by diagonally hatched regions, corresponding to one and two statistical standard deviations. The individual indirect constraints lie between the pairs of solid lines.

is different from the phase of the tree amplitude. Therefore the measured time-dependent asymmetry, proportional to $\sin(\Delta m \Delta t)$, is not equal to $\sin 2\phi_2$ but instead will have a large unknown correction. The presence of the extra contribution also induces an additional time-dependent term proportional to $\cos(\Delta m \Delta t)$. This is called penguin pollution.

In addition, there are a number of other purely experimental complications that were not present in the $\sin 2\phi_1$ measurement. The branching fraction for the $B^0 \rightarrow \pi^+\pi^-$ decay is small, $\mathcal{B}(B \rightarrow \pi^+\pi^-) = (4.4 \pm 0.9) \times 10^{-6}$ (35). Moreover, even after the application of high-momentum particle identification requirements, the small CP eigenstate signal is expected to have a large continuum background.
Despite the difficulties, the first measurements of the effective value of $\sin 2\phi_2$ using the $\pi^+\pi^-$ mode have been reported (41, 42). The results are somewhat controversial, and in contrast to the $\sin 2\phi_1$ measurements, the agreement between the two experiments is marginal. BABAR reports $S_{\pi\pi} = \lambda \sin 2\phi_2(\text{eff.}) = 0.02 \pm 0.34(\text{stat.}) \pm 0.05(\text{sys.})$, whereas Belle finds $S_{\pi\pi} = \lambda \sin 2\phi_2(\text{eff.}) = -1.23 \pm 0.41(\text{stat.})^{+0.08}_{-0.07}(\text{sys.})$.

Several methods to measure $\phi_3$ or angles related to it have been proposed. These include measurements of branching fractions of $B$ decays with no charm in the final state (43) or of $B \to D^{(*)}K^{(*)}$ (44), and measurements of the time-dependent asymmetry of $B \to D^{(*)}\pi$ (45), $B \to DK\pi$ (46), and $B \to DK_0^* S$ (47). All these methods are being pursued, although the current data sample is not large enough to yield significant results.

In addition to the program of measuring the angles of the unitarity triangle, there is also the question of whether there are additional CP-violating phases from new interactions or physics beyond the standard model. At the moment, such new phases are poorly constrained. One way to attack this question is to measure the time-dependent CP asymmetry in penguin-dominated modes such as $B^0 \to \phi K^0_S$ or $B^0 \to \eta' K^0_S$, where heavy new particles may contribute to the loop diagram, for comparison with the asymmetry in $B^0 \to J/\psi K^0_S$ and related charmonium modes. In the absence of new physics, the asymmetries in the charmonium and penguin-decay modes should be equal. However, if new-physics contributions are present in penguin loops, these asymmetries will differ substantially (48). It is also important to verify that the CP asymmetries in other tree-level quark-level processes, such as $b \to c\bar{c}d$ transitions, are the same as those in $b \to c\bar{c}s$. Examples of decay modes in which such tests can be performed are $B \to D^{**+}D^{-}$ and $B \to J/\psi\pi^0$.

An order of magnitude more data are needed for stringent tests (49, 50). This search for new phenomena in CP violation will be one of the most interesting aspects of the next phase in $B$-factory physics.

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CONTENTS

FRONTISPIECE, Ernest D. Courant xii
ACCELERATORS, COLLIDERS, AND SNAKES, Ernest D. Courant 1
SEMILEPTONIC HYPERON DECAYS, Nicola Cabibbo, Earl C. Swallow, and Roland Winston 39
TESTS OF DISCRETE SYMMETRIES WITH CPLEAR, Philippe Bloch and Ludwig Tauscher 123
LATTICE QCD AT FINITE TEMPERATURE, E. Laermann and O. Philipsen 163
OBSERVATION OF THE TAU NEUTRINO, B. Lundberg, K. Niwa, and V. Paolone 199
DIRECT REACTIONS WITH EXOTIC NUCLEI, P.G. Hansen and J.A. Tostevin 219
DEVELOPMENTS AND APPLICATIONS OF HIGH-PERFORMANCE CCD AND CMOS IMAGING ARRAYS, James Janesick and Gloria Putnam 263
TOP-QUARK PHYSICS, Dhiman Chakraborty, Jacobo Konigsberg, and David Rainwater 301
ESTABLISHMENT OF CP VIOLATION IN B DECAYS, T.E. Browder and R. Faccini 353
HIGH-CURRENT ENERGY-RECOVERING ELECTRON LINACS, Lia Merminga, David R. Douglas, and Geoffrey A. Krafft 387
D^{0}–D^{0} MIXING AND RARE CHARM DECAYS, Gustavo Burdman and Ian Shipsey 431

INDEXES
Cumulative Index of Contributing Authors, Volumes 44–53 501
Cumulative Index of Chapter Titles, Volumes 44–53 504

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