B^0-\bar{B}^0 mixing and violations of CP symmetry

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In view of a possible long lifetime for B mesons, \( \tau_B \sim 10^{-12} \) sec, we reexamine predictions of the standard model for B^0-\bar{B}^0 mixing. We estimate theoretical uncertainties in the computation of the relevant matrix element by comparing the vacuum-saturation approximation with predictions obtained in the bag model and in the harmonic-oscillator model with and without relativistic corrections. For the B_d-B_{\bar{d}} system we find mixing leading to a like-sign dilepton yield of at most a few percent of the opposite-sign dilepton rate; B_d-B_{\bar{d}} mixing should be stronger. While very little CP violation is expected in B^0-\bar{B}^0 mixing, we find that certain CP asymmetries in on-shell B_d decays could reach a level of 10% or more.

I. INTRODUCTION

For various reasons it is appropriate to reanalyze the prospects for B^0-\bar{B}^0 mixing and the observation of CP violation in B decays:

(a) The statistics that will be accumulated on B decays in the near future will allow B^0-\bar{B}^0 mixing and CP violations in these systems to be subjected to experimental scrutiny.

(b) The theoretical analysis of B^0-\bar{B}^0 mixing has so far employed arguments based largely on simplicity, such as vacuum saturation of matrix elements, while ignoring other tools available in our theoretical arsenal to calculate matrix elements, such as the bag model or the harmonic-oscillator model.

(c) There are two new pieces of experimental information that are of direct relevance to such an analysis: an improved upper bound on the ratio of \( b \rightarrow u \) over \( b \rightarrow c \) transitions,

\[
\frac{\Gamma(b \rightarrow u)}{\Gamma(b \rightarrow c)} < 0.05 ,
\]

and a first tentative measurement of the \( B \)-meson lifetime, \( \tau(B) \sim 10^{-12} \) sec.\(^1\)

In terms of the Kobayashi-Maskawa angles these numbers can conveniently be reexpressed as

\[
s_2^2 + s_3^2 + 2s_2s_3\cos\delta \approx 4.2 \times 10^{-3} B(b \rightarrow c) \left( \frac{10^{-12} \text{sec}}{\tau_B} \right) ,
\]

(1.2)

Thus, it is suggested that both \( s_2 \) and \( s_3 \) are considerably smaller than \( s_1 \approx 0.23 \). If true, this would lead to some interesting phenomenological consequences which will be discussed later.

The paper will be organized as follows: In Sec. II we make a few short comments on the calculation of \( \mathcal{L}_{\text{eff}} \), the effective interaction responsible for B^0-\bar{B}^0 mixing. In Sec. III we discuss various methods to compute or at least estimate the matrix element \( \langle B^0 | \mathcal{L}_{\text{eff}} | \bar{B}^0 \rangle \) and the amount of mixing. Section IV contains an analysis of how much CP violation might be expected in B^0-\bar{B}^0 mixing and in B decays. Finally, in Sec. V we present our conclusions.

II. CALCULATION OF \( \mathcal{L}_{\text{eff}}(\Delta B = 2) \)

The mass matrix determining B^0-\bar{B}^0 mixing is calculated by computing the well-known box diagrams. Including strong radiative corrections one finds

\[
\mathcal{L}_{\text{eff}}^{\text{box}} = \eta_{\text{QCD}} \left[ \frac{G_F}{4\pi} \right]^2 \left[ \xi^2 t^2 f^2 + 2s_2s_3 \cos\delta \approx 4.2 \times 10^{-3} B(b \rightarrow c) \left( \frac{10^{-12} \text{sec}}{\tau_B} \right) \right] (\bar{d}q)_{\nu-A} (\bar{d}q)_{\nu-A} \quad \text{(2.1)}
\]

with \( \xi^2 = -s_1s_2e^{i\phi} \) in the small-angle approximation and

\[
\eta_{\text{QCD}} = \left[ \frac{\alpha_s(m_b)}{\alpha_s(m_t)} \right]^{-6/23} \left[ \frac{3}{2} \frac{\alpha_s(m_t)}{\alpha_s(M_W)} \right]^{-4/7} \left[ \frac{\alpha_s(m_t)}{\alpha_s(M_W)} \right]^{2/7} + \frac{1}{2} \frac{\alpha_s(m_t)}{\alpha_s(M_W)} \left[ \frac{\alpha_s(m_t)}{\alpha_s(M_W)} \right] \quad \text{(2.1)}
\]

\[s_2^2 \approx 3.9 \times 10^{-2} B(b \rightarrow u) \left( \frac{10^{-12} \text{sec}}{\tau_B} \right).
\]

With \( B(b \rightarrow u) \approx 0.05 \) and \( \tau_B \sim 10^{-12} \) one finds

\[s_3 \approx 5 \times 10^{-2}.
\]

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b (q) stands for the bottom- (down- or strange-) quark field. At short distances the effective $\Delta B = 2$ coupling can be approximated by the box diagram. Such a procedure has first been applied to $K^0\bar{K}^0$ mixing and it turned out to be rather successful in the sense that it leads to a correct prediction for the charm-quark mass. However, there are bound to be contributions to $\mathcal{L}_{\text{eff}}(\Delta S = 2)$ which are not determined by short-distance dynamics, namely, $K^0\bar{K}^0$ transitions that proceed via virtual $\pi$, $\eta$, $\pi\pi$, etc., intermediate states. It is not known yet how to compute such contributions in a reliable fashion. We will come back to this complication later.

The $B^0\bar{B}^0$ transition operator, on the other hand, should be determined basically by short-distance dynamics, since $M_B$ is so much heavier than typically hadronic mass scales; for the same reason one expects the spectator mechanism to yield a very good description of $B$ decays. The fact that $\Gamma(b \to c)$ is much larger than $\Gamma(b \to u)$ strengthens the dominance of short-distance dynamics even more since pionic intermediate states will be highly suppressed. Therefore,

$$\mathcal{L}_{\text{eff}}(\Delta B = 2) \approx \mathcal{L}_{\text{eff}}^{\text{HO}}(\Delta B = 2) .$$ (2.2)

### III. ESTIMATES OF $\langle B^0 | L_{\text{eff}}(\Delta B = 2) | B^0 \rangle$ AND THE AMOUNT OF MIXING

The presumably gravest uncertainties arise when one tries to calculate the appropriate matrix element of $\mathcal{L}_{\text{eff}}$ since that is certainly not in the realm of short-distance physics.

In our subsequent discussion we will use the following definition:

$$\mathcal{M} = \frac{1}{m_B} \langle \bar{b}q \rangle_{\nu-A} \langle \bar{b}q \rangle_{\nu-A} | B^0 \rangle$$

$$= \frac{4}{9} R_B f_B^2 m_B .$$ (3.1)

Saturating the matrix element (3.1) by inserting just the vacuum state yields $R_B = 1$, where $f_B$ is the decay constant of the $B$ meson. This is the highly popular vacuum-saturation approximation (VSA).

The decay constant $f_B$ can be measured in principle via the decay $B \to \pi \nu$, but at present it is not known. Theoretical estimates in the literature range between 100 and 500 MeV. This uncertainty, which is already quite unfortunate, is further compounded by our ignorance concerning $R_B$, i.e., the numerical quality of the VSA.

It is tempting to use a value for $R_B$ as extracted in the $K^0\bar{K}^0$ system. There are, however, some drawbacks to such a procedure:

(i) Since we do not know of a convincing argument for equating $\mathcal{L}_{\text{eff}}(\Delta S = 2)$ with $\mathcal{L}_{\text{eff}}^{\text{HO}}(\Delta S = 2)$, we cannot rely on $M(K_q) - M(K_{\bar{q}})$ for fixing $R$.

(ii) Much better arguments can be given for CP violation to be determined by short-distance dynamics. Yet to obtain $\epsilon_K$ one has to know the appropriate on-shell matrix element of the box operator for which a short-distance analysis does not suffice. A priori there is no clear reason why $R_b$ parametrizing this matrix element in $K^0$ physics should be equal to $R_B$.

(iii) The interesting suggestion has been made to derive $R$ from the observed $K^+ \to \pi^+ \pi^0$ width via current algebra or chiral perturbation theory. Such an argument is, however, not expected to work for the much heavier $B$ meson—another reason why the parameter $R_B$ could be quite different in the two cases.

Therefore, we will estimate $\langle B^0 | L_{\text{eff}} | B^0 \rangle$ and its theoretical uncertainties by calculating it directly using phenomenological hadronic wave functions. Of these wave functions we demand of course that they give at least a decent fit to nonstrange and strange hadrons.

We will actually employ three different models, each stressing different aspects and thus hopefully complementing each other: (A) the non-realistic harmonic-oscillator (HO) model, (B) the relativistically corrected HO(RHO) model, and (C) the bag model.

Although no clear reason can be given why the HO and the RHO models should work in the first place, one does not expect them to give a poorer description of bottom states than of strange states. As far as a bag model, where hadrons are taken as a sphere, is concerned, it has even been suggested to provide a more reliable description for mesons containing one—but not two—heavy quarks. Our discussion of cases (A) and (B) parallels the treatment of $K^0\bar{K}^0$ mixing in Ref. 8.

#### A. The HO model

The harmonic-oscillator quark model of Isgur and Karl ignores all relativistic effects and therefore has to be considered as a phenomenological ansatz, however, as an amazingly successful one.

The matrix element [Eq. (3.1)] is found to be given by

$$\mathcal{M}^{\text{HO}}_{\langle bq \rangle} = 16 \left( \frac{\beta_{bq}^2}{\pi} \right)^{3/2} ,$$ (3.2)

$$\beta_{bq}^2 = \frac{2 m_b m_q x}{m_b + m_q}$$ (1/2),

where $x$ describes the coupling strength of the HO potential.

Fitting the model yields $x \sim 10^{-2} \text{ GeV}^3$. With the (constituent) quark masses $M_d = \frac{1}{3} \text{ GeV}$, $M_s = 0.5 \text{ GeV}$, and $M_b = 5 \text{ GeV}$ one obtains

$$\mathcal{M}^{\text{HO}}_{B_d} \approx 6.4 \times 10^{-2} \text{ GeV}^3 ,$$ (3.4)

$$\mathcal{M}^{\text{HO}}_{B_s} \approx 8.5 \times 10^{-2} \text{ GeV}^3 .$$ (3.5)

#### B. The RHO model

Colic et al. include some relativistic corrections to the HO model by replacing Pauli spinors by Dirac spinors. They find

$$\mathcal{M}^{\text{RHO}}_{B} = 16 \left[ 1 - \frac{6 \beta_B^2}{(E_b + m_p)(E_q + m_q)} \right]$$

$$+ \frac{9 \beta_B^4}{(E_b + m_p)^2(E_q + m_q)^2} .$$ (3.6)
with
\[ \hat{g} = \alpha_B \left( \frac{\beta_B}{\pi} \right)^{3/2}, \]
\[ \alpha_B = \frac{1}{4E_b E_q} (E_b + m_b)(E_q + m_q), \] (3.7)
\[ E_i = (m_i^2 + \frac{3}{2} \beta_i R^2)^{1/2}. \]

Inserting the same mass values as before we obtain
\[ a = N_b^2 N_q^2 \int_0^R dr r^2 \left[ j_0^2 \left( \frac{\xi_i r}{R} \right) j_0^2 \left( \frac{\xi_b r}{R} \right) + e_b^2 e_q^2 j_1^2 \left( \frac{\xi_i r}{R} \right) j_1^2 \left( \frac{\xi_b r}{R} \right) \right], \] (3.14)
\[ b = N_b^2 N_q^2 \int_0^R dr r^2 \left[ e_b j_0^2 \left( \frac{\xi_i r}{R} \right) j_1^2 \left( \frac{\xi_b r}{R} \right) + e_q^2 j_0^2 \left( \frac{\xi_i r}{R} \right) j_1^2 \left( \frac{\xi_b r}{R} \right) \right], \] (3.15)
\[ c = N_b^2 N_q^2 \int_0^R dr r^2 e_b e_q j_0 \left( \frac{\xi_i r}{R} \right) j_1 \left( \frac{\xi_i r}{R} \right) j_0 \left( \frac{\xi_b r}{R} \right) j_1 \left( \frac{\xi_b r}{R} \right). \] (3.16)

The \( j_i \) are spherical functions; the \( \xi_i \) represent momenta in units of the natural scale \( R^{-1} - \beta_i = \xi_i r^{-1} \), and are determined to be roots of the equation
\[ \tan(\xi_i) = \frac{\xi_i}{1 - m_i R - (\xi_i^2 + m_i^2 R^2)^{1/2}}. \] (3.17)

For \( d \) quarks with \( M_d \gtrsim 0 \) one finds \( \xi_d \approx 2.0431 \) while \( s \) quarks with \( M_s \approx 280 \) MeV imply \( \xi_s \approx 2.43 \); \( b \) quarks, on the other hand, with \( M_b \approx 4.6-4.8 \) GeV are so heavy that they can safely be treated nonrelativistically.\(^7\) This limit is obtained by letting \( m_b R \) go to infinity; then \( \xi_b = \pi \) and simplifications occur (e.g., \( c = 0 \) since \( \xi_b = 0 \)). We obtain
\[ M_{\text{bag}}^{\text{HO}} \approx 3.9 \times R^{-3} \approx 6 \times 10^{-2} \text{ GeV}^3, \] (3.18)
\[ M_{\text{bag}}^{\text{RHO}} \approx 5.5 \times R^{-3} \approx 8.5 \times 10^{-2} \text{ GeV}^3, \]

with \( R^{-1} \approx 0.25 \) GeV.\(^7\)

Instead of \( m_b R \rightarrow \infty \) one can use \( m_b \approx 5 \) GeV in which case \( \xi_b \approx 3.064 \). Then one finds
\[ M_{\text{bag}}^{\text{HO}} \approx 5.2 \times 10^{-2} \text{ GeV}^3, \]
\[ M_{\text{bag}}^{\text{RHO}} \approx 7.7 \times 10^{-2} \text{ GeV}^3, \]
i.e., very little difference from (3.17) and (3.18). This illustrates our statement that the bag-model results are not very sensitive to the choice of the \( b \)-quark mass.

In Table I we have summarized our results obtained so far and have compared them to the results of the vacuum-saturation approximation (VSA).

Some comments are in order:
(i) The three models, in particular, the harmonic-oscillator model and the bag model, yield very similar numerical results although the light antiquark is treated in a very different fashion, namely, relativistically in the bag model and nonrelativistically in the HO model. This agreement does not hold for the \( K^0, \bar{K}^0 \) case.

(ii) The value of \( f_B = 150 \) MeV was picked somewhat arbitrarily to illustrate the order of magnitude of the VSA result. Potential models\(^1\) yield \( f_B \approx 125 \) MeV, \( f_B \approx 175 \) MeV, while a bag-model calculation\(^1\) gives \( f_B \approx 100 \) MeV. Thus, one estimates
\[ M_{\text{bag}}^{\text{VSA}} \approx (7-11) \times 10^{-2} \text{ GeV}^3, R_B \approx 0.5-0.8. \]

(iii) The bag-model result is fairly stable under variation of parameters and positive in sign. This is in marked contrast to the \( K^0, \bar{K}^0 \) case where small variations in the bag parameters affect the magnitude of \( M_K \) drastically and can even change the sign.\(^8\) For completeness it should be kept in mind that the spherical bag model being used here will cease to offer a reasonable description when the antiquark becomes too heavy.

### Table I. Values of \( M_B \) (in units of \( 10^{-2} \) GeV³), calculated in the VSA, the HO model, the RHO model, and the bag model.

<table>
<thead>
<tr>
<th>( B_d )</th>
<th>( M_B )</th>
<th>( \frac{f_B}{150 \text{ MeV}} )</th>
<th>( \text{HO} )</th>
<th>( \text{RHO} )</th>
<th>( \text{bag} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( B_d )</td>
<td>15.8</td>
<td>( f_B )</td>
<td>6.4</td>
<td>5.1</td>
<td>5.2</td>
</tr>
<tr>
<td>( B_s )</td>
<td>15.8</td>
<td>( f_B )</td>
<td>8.5</td>
<td>7.2</td>
<td>7.7</td>
</tr>
</tbody>
</table>
(iv) One should recall that in nonrelativistic models the decay constant \(f_M\) of a meson with mass \(M\) is given by the wave function at the origin,

\[
\frac{f_M^2}{M} = \frac{12 |\psi(0)|^2}{M},
\]

and therefore

\[
\mathcal{M} = \frac{3}{4} R_B f_M^2 M = 16 R_B |\psi(0)|^2.
\]

Thus, a nonrelativistic treatment suggests that \(\mathcal{M}\) has the same value for all mesons with the same reduced mass \(\mu\).

\[
M_{12} = \left(\frac{G_F}{4\pi}\right)^2 \mathcal{M} \eta_{QCD} \left[ \xi_i^2 \left( m_i^2 + \frac{1}{2} m_b^2 + \frac{1}{2} m_b^2 \ln \frac{m_i^2}{m_b^2} \right) + O \left( \frac{m_b^2}{m_i^2} \right) \right].
\]

For \(B_d \equiv (\bar{b}d)\) mesons we then find with \(\xi_i^2 \approx s_2^2 e^{2i\delta}\)

\[
x_{B_d} \approx 13 s_2^2 \cos 2\delta \left[ \frac{\mathcal{M}}{5 \times 10^{-2} \text{ GeV}} \right] \left[ \frac{\tau_B}{10^{-12} \text{ sec}} \right]
\]

where we have set \(m_1 \sim 35\) GeV. To a first approximation \(x_{B_d}\) scales like \((m_1/35\) GeV\(^2\). In Ref. 2 it was shown that for \(\tau_B \approx 10^{-12}\) sec, \(m_1 \approx 35\) GeV (and \(R_K \approx 0.37\)) one can derive a lower limit on \(|s_2 s_3 \sin \delta|\) from the measured value of \(\epsilon_K\) and \(M(K_L) - M(K_S)\):

\[
|s_2 s_3 \sin \delta| \geq 2.2 \times 10^{-3} \text{ if } \cos \delta < 0.
\]

Even accepting all these values does not suffice yet to fix all the relevant parameters; however, upper bounds can be given, such as \(s_2 \leq 0.1\) from Eqs. (1.2) and (1.4) and \(\cos \delta \leq \frac{1}{2}\) from Eq. (3.25). Assuming these upper bounds to be saturated we find as an order-of-magnitude estimate

\[
x_{B_d} \approx 0.13 \left( \frac{\mathcal{M}}{5 \times 10^{-2} \text{ GeV}^3} \right).
\]

Thus, the VSA yields \(x_{B_d} \sim 0.4\) while the bag-model ansatz leads to \(x_{B_d} \sim 0.14\).

For \(B_s = (\bar{s}s)\) mesons we find much larger values. Since \(\xi_i^2 \approx (s_2^2 + s_3^2 e^{2i\delta})^2\) in that case we get

\[
x_{B_s} \approx 260 F \left[ \frac{\mathcal{M}}{5 \times 10^{-2} \text{ GeV}^3} \right] \left[ \frac{\tau_B}{10^{-12} \text{ sec}} \right],
\]

for \(\tau_B \approx 10^{-12}\) sec one can write

\[
F \approx 4.2 \times 10^{-3} - 2 s_3^2 \sin^2 2\delta.
\]

Again no firm prediction of \(F\) and thus of \(x_{B_s}\) can be given at present. Saturating the lower bound on \(|s_2 s_3 \sin \delta|\) derived from Eqs. (1.4) and (3.25) one finds \(F \sim 10^{-3}\) and therefore

\[
x_{B_s} \sim 0.3 \left( \frac{\mathcal{M}}{5 \times 10^{-2} \text{ GeV}^3} \right).
\]

Thus, the VSA gives \(x_{B_s} \sim 1\) while the bag model yields \(x_{B_s} \sim 0.5\) and increases with \(\mu\)—as borne out by comparing \(\mathcal{M}_{K^0}^{\pi^0}\) and \(\mathcal{M}_{\pi^0}^{K^0}\).

To summarize our analysis so far: using the VSA for computing the matrix element \(\mathcal{M}_B\) with \(f_B \approx 150\) MeV might lead to an overestimate by a factor of roughly 2.

The amount of mixing is usually expressed in terms of

\[
x = \frac{2 \Delta m}{\Gamma},
\]

where

\[
\Delta m = m_1 - m_2 = 2 \Re M_{12},
\]

and \(B^0 \rightarrow \bar{B}^0\) mixing will lead to like-sign dileptons in \(e^+ e^-\) annihilation. Since the \(B^0 \rightarrow \bar{B}^0\) pair is produced in a coherent quantum state one has to include effects due to Bose statistics. If \(B^0 \rightarrow \bar{B}^0\) are produced in a p-wave state, as happens on \(\gamma(4s)\), one finds for the dileptons from \(B^0 \rightarrow \bar{B}^0\) decays

\[
R_l = \frac{N(l^+ l^+)+N(l^- l^-)}{N(l^+ l^-)} = \frac{x^2}{2 + x^2}.
\]

With the numbers given above we find

\[
R_l \approx (B_1) \approx 0.01 - 0.08,
\]

\[
R_l \approx (B_2) \approx 0.10 - 0.30.
\]

For \(l = \text{relative orbital angular momentum} = \text{even as in } e^+ e^- \rightarrow B^* \bar{B} \rightarrow B \bar{B} \gamma\) one obtains much larger values:

\[
R_l \approx (B_1) \approx 3 x^2 + x^4 \frac{0.04 - 0.23}{2 + x^2 + x^4},
\]

\[
R_l \approx (B_2) \approx 0.30 - 1.
\]

We stress again that these numbers are not firm predictions, since the relevant parameters are not sufficiently well known yet; they are given to illustrate the order of magnitude of such effects and their inherent theoretical uncertainties.

**IV. CP VIOLATION IN B-MESON TRANSITIONS**

It has to be kept in mind that CP violation can surface in \(B\)-meson transitions in two different ways: (a) it can occur in \(B^0 \rightarrow \bar{B}^0\) mass mixing as it occurred in \(K^0 \rightarrow \bar{K}^0\) mixing where it is characterized by the quantity \(\epsilon_K\). (b) It can appear also in on-shell \(B\) decays in analogy to the quantity \(\epsilon\) defined in \(K\) decays.

(i) It has been pointed out before that chances to observe CP violation in \(B^0 \rightarrow \bar{B}^0\) mixing are rather slim. The relevant parameters are defined as follows:

\[
\frac{\Gamma(B^0 \rightarrow l^+ \nu X)}{\Gamma(B^0 \rightarrow l^- \bar{\nu} X)} = \frac{1 - \epsilon_B}{1 + \epsilon_B} \left( \frac{\Delta m^2}{2 \Gamma^2 + (\Delta m^2)^2} \right),
\]

where

\[
\Delta m = m_1 - m_2 = 2 \Re M_{12},
\]

and

\[
\epsilon_B = \frac{3 x^2 + x^4}{2 + x^2 + x^4} \approx 0.04 - 0.23.
\]
\( r = \frac{\Gamma(B^0 \to l\bar{\nu}X)}{\Gamma(B^0 \to l^+\nu X)} \)
\[
= \frac{1 + \epsilon_B}{1 - \epsilon_B} \frac{(\Delta m)^2 + \frac{1}{2}(\Delta \Gamma)^2}{2\Gamma^2 + (\Delta m)^2 - \frac{1}{4}(\Delta \Gamma)^2} .
\]

(4.2)

With these quantities one can express the total lepton charge asymmetry in semileptonic decays of the \( B^0\bar{B}^0 \) system:
\[
A_f \equiv \frac{N(l^+)-N(l^-)}{N(l^+)+N(l^-)} = \frac{r - \bar{r}}{2 + r + \bar{r}} .
\]

(4.3)

It is a general feature of the standard six-quark model that \( A_f \) is very small, independent of the values of \( s_2 \), \( s_3 \), and \( m_l \): \( A_f < 10^{-2} \). Small values of \( s_2 \) and \( s_3 \) and \( m_l \geq 30 \) GeV will actually decrease \( A_f \) much further: \( A_f \sim 10^{-4} \sim 10^{-3} \).

(ii) From this pessimistic estimate of \( CP \) violation in \( B^0\bar{B}^0 \) mixing one should not infer that \( CP \) violation will be unobservable in \( B \) decays. The pattern of \( CP \) violation in the \( B^0 \) system could well be completely different from the one in the \( K^0 \) system: on-shell \( B \) decays could show sizable \( CP \) asymmetries as explained in Refs. 12 and 13. Here we will reanalyze those predictions.

The main idea is that there are certain final states \( f \) into which both \( B^0 \) and \( \bar{B}^0 \) can decay—possibly after a multistep reaction:

\[
B^0 \rightarrow f
\]

Candidates for such a final state are

\[
B_d \rightarrow D^0 + \pi^0 s \]
\[
\bar{B}_d \rightarrow \bar{D}^0 + \pi^0 s \]
\[
B_d \rightarrow D\bar{D} K_s + \pi^0 s .
\]

Mixing is evoked to provide interference of two amplitudes thus exposing possible complex phases in them; however, \( CP \) violation in the mixing itself is not required. One can define a \( CP \) asymmetry in \( e^+e^- \rightarrow B^0\bar{B}^0 + X \),

\[
A = \frac{\sigma(l^+\bar{X}f) - \sigma(l^-\bar{X}f)}{\sigma(l^+\bar{X}f) + \sigma(l^-\bar{X}f)} = -\frac{2xu^2\sin2\phi}{1+y^2+y^2\cos2\phi} ,
\]

(4.4)

if \( B^0\bar{B}^0 \) are produced in a charge-conjugation-even state, like \( e^+e^- \rightarrow B^0\bar{B}^0 + \gamma \); it vanishes otherwise. Here we have used the notation
\[
x = \frac{2m}{\Gamma}, \quad y = \frac{\Delta \Gamma}{\Gamma}, \quad a = \frac{1-y^2}{1+x^2} ,
\]
\[
-\epsilon = e^{-2i\phi} \frac{pM}{qM}, \quad \frac{p}{q} = \frac{1+\epsilon_B}{1-\epsilon_B} ,
\]
\[
M = \langle f | \mathcal{L}(\Delta B = 1) | B^0 \rangle ,
\]
\[
\bar{M} = \langle f | \mathcal{L}(\Delta B = 1) | \bar{B}^0 \rangle .
\]

The cases of \( B_d \) and \( B_s \) decays have to be treated separately; first we consider the \( B_d\bar{B}_d \) system:

\[
\sin2\phi_d = \frac{s_1^2 \sin2\delta + 2s_2s_3 \sin\delta}{s_2^2 + s_3^2 + 2s_2s_3 \cos\delta} = \frac{2s_3 \sin(\delta (s_3 \cos\delta + s_3))}{s_2^2 + s_3^2 + 2s_2s_3 \cos\delta} .
\]

(4.6)

It is again premature to give a firm prediction for \( \sin2\phi_d \), but using "typical" values for \( s_2, s_3 \), and \( \delta \) that satisfy the relations (1.2), (1.4), and (3.25) one finds as a reasonable estimate \( \sin2\phi_d \sim \frac{1}{5} \), and therefore

\[
|A_d| \sim 11-30\% .
\]

(4.7)

Of course it will be very hard to analyze special exclusive decay modes and still accumulate sufficient statistics. Instead one can compare inclusive modes like \( B^0\bar{B}^0 \rightarrow l^-K_S X \) vs \( B^0\bar{B}^0 \rightarrow l^+K_S X \) or \( B^0\bar{B}^0 \rightarrow l^-l^+D X \) vs \( B^0\bar{B}^0 \rightarrow l^-l^+D X \) where the second lepton comes from D decays. When performing such a semi-inclusive analysis one will certainly include channels with no \( CP \) asymmetry and thus dilute the effect. However, in view of the possible size of \( A_f \) [Eq. (4.7)] this is a viable option. The prospects are much more gloomy for the \( B_d\bar{B}_d \) system since one finds within the standard six-quark model

\[
\sin2\phi_d = 0 + O\left( \frac{m_c}{m_t} \right)^2 s_3 \] .

(4.8)

V. SUMMARY

The neutral \( B \) mesons offer a great opportunity to study complex phenomena such as \( B^0\bar{B}^0 \) mixing and \( CP \) violation.

A long \( B \)-meson lifetime \( \sim 10^{-12} \) sec will certainly increase the observability of such effects. Numerical predictions for the relevant quantities are hampered by uncertainties in the size of certain matrix elements. We estimated these uncertainties to amount to a factor of roughly 3. While we consider these model predictions to be more reliable for \( B^0\bar{B}^0 \) than for \( K^0\bar{K}^0 \) matrix elements, we do not see a convincing argument for extrapolating experimental information on the \( K^0\bar{K}^0 \) system to the \( B^0\bar{B}^0 \) system.

As far as mixing is concerned, we expect some mixing to occur for \( B_d\bar{B}_d \), leading to a like-sign dilepton rate of at most a few percent of the opposite-sign dilepton yield. For \( B_s\bar{B}_s \), on the other hand, we expect very sizable mixing.

\( CP \) violation in \( B^0\bar{B}^0 \) mixing will be very hard to observe if the standard six-quark model is correct. Very sizable \( CP \) asymmetries might, however, appear in on-shell \( B_d \) decays. This could be studied in \( e^+e^- \) annihilation just above the \( B\bar{B}^* \) threshold or on the \( Z^0 \) resonance.

Note added: Our numbers for the expected size of \( B^0\bar{B}^0 \) mixing are somewhat smaller than those found in Ref. 14, where the VSA was employed.

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1S. Stone, invited lecture at the Lepton-Photon Symposium, Cornell University, 1983 (unpublished).


10R. E. Shrock and S. B. Treiman, Phys. Rev. D 19, 2148 (1979); B. McWilliams and O. Shankar, ibid. 22, 2853 (1980). Our sign differs from that of these authors and is in agreement with that of Ref. 8. To support our statement we mention that the sign of the VSA result should agree with the axial-axial contribution in the bag model. See, for example, H. Krasemann, Phys. Lett. 96B, 397 (1980).

