which they might be distinguished from the known particles? Since the mass difference $\Delta$ cannot be estimated at present, we can only speculate about possible decay modes of $p_2$. It might, e.g., decay by a $\beta$ interaction: $p_2 \rightarrow n_1 + e_{1,2}^- + \nu$. Other weak decay modes might exist; in particular, one or more $\pi$ mesons or $K$ mesons can be emitted if $\Delta$ were sufficiently high. Since $p_2$ must be assumed to have also strong interactions, though not in decay, it may occur bound to other, ordinary, nucleons, and its decay in such a state may resemble a hyperfragment disintegration. The same holds for $\tilde{p}_2$, which could either annihilate "slowly" inside a nucleus through its weak interaction, e.g., $\tilde{p}_2 + n_1 \rightarrow e_{1,2}^- + \bar{\nu}$, or first decay into $\tilde{p}_1$ or $\tilde{n}_1$ with consequent rapid annihilation.

I wish to thank G. Feinberg, F. Gursey, and M. Gell-Mann for interesting discussions.

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Roundtable discussion on $\pi$-decay (Tollestrup, Steinberger, Miller, Anderson, and Pollak) at Gatlinburg Conference on Weak Interactions, October 1958, Revs. Modern Phys. (to be published).


6From time to time there have been reports of cosmic-ray events observed in photographic emulsions which appear to show a $Q$ value larger than expected from hyperon decay. In the light of the question raised above, some of these events might be worth re-analyzing. See, e.g., Y. Eisenberg, Phys. Rev. 26, 541 (1954); Fry, Schneps, and Swami, Phys. Rev. 97, 1189 (1955), and Nuovo cimento 2, 346 (1955); Castagnoli, Cortini, and Frantinetti, Nuovo cimento 2, 550 (1955).

**RADIATIVE CORRECTIONS TO PION BETA DECAY**

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Recent experiments have shown that the ratio $R_0$ of the rates $\Gamma_e, \Gamma_\mu$ for $\pi^-e^-+\nu$ and $\pi^-\mu^-+\nu$ modes of decay in the neighborhood of the value predicted by the universal beta-decay theory, i.e., $R_0=(12.78 \times 10^{-5})$. However, to compare the results of a precise experiment with the predicted number the radiative corrections should be included in the theoretical estimate of the ratio. The corrections do not cancel in the ratio since they depend on the electron and muon masses and, furthermore, produce a surprisingly large correction to the ratio.

Using the universal $(V,A)$ theory we express the general matrix element for the process $\pi^-(e \text{ or } \mu)+\nu+\gamma$ to order $e$ as

$$M_\nu=\left[f_1^0 \mu \nu + f_2 k \mu \rho \nu + f_3 k \rho \mu \rho \nu \right] \times \bar{\psi}_{l_2} \gamma_{l_2} (1+i\gamma_5) \psi_{l_2} \ldots ,$$

where $p$ and $k$ are the momentum of the pion and photon, respectively, and where each of the covariants $f_1, f_2, f_3$ depend on the scalars $m_p^2, p \cdot k, k^2$. In addition to $M_\nu$ we also consider the matrix elements which arise from bremsstrahlung photons. In writing Eq. (1) we have used the Lorentz condition to eliminate terms such as $k p_\mu k \mu$. We note that gauge invariance implies that $f_1$ at $k=0$ is determined by the amplitude for $\pi^- (e \text{ or } \mu) + \nu$.

In the phenomenological theory with direct $\pi^-(e \text{ or } \mu)$ coupling only $f_1$ would be present in Eq. (1). In a realistic theory involving virtual nucleon loops, the other covariants $f_2, f_3$ would be expected to enter. If we consider the decay of the pion to occur through one virtual nucleon pair then we find that $f_{2,3} \sim (1/M^2) f_1$ where $M$ is the nucleon mass and that in the limit of very large nucleon mass $f_1$ does not depend on $p \cdot k$ and $k^2$. If we consider $(k^2/M^2) \ll 1$ then application of the Ward identity leads to the same conclusions to all orders in the pion-nucleon coupling. We therefore have neglected $f_{2,3}$ in the calculation of the radiative corrections, and have assumed $f_1$ is independent of $k$. In this case $f_1$ will cancel out in the ratio of the rates.

Using the first term of Eq. (1) we have calculated the radiative corrections, in the usual manner, by including all possible real and virtual electromagnetic processes to order $e^2$. Since the inclusion of the inner bremsstrahlung changes the number of particles in the final states from two to three there is now a spectrum of energies available to the electron or muon. If we ask how the ratio of rates is affected for electrons or muons having an energy near the maximum energy and suffering an energy loss less than...
\[ \Delta E, \text{ then we obtain for the new ratio } R, \text{ in units of } \hbar = c = 1 \text{ and } e^2 = 1/137, \]
\[ R = (12.78 \times 10^{-8}) \left[ 1 - \frac{2e^2}{\pi} \left\{ \frac{1}{4} \ln \left( \frac{m_\pi/m_\mu}{m_\pi/m_\mu} \right) \right. \right. \]
\[ + \ln(2\Delta E/m_\pi) - \ln(2\Delta E/m_\mu) + \left( \frac{m_\pi^2 + m_\mu^2}{2\Delta E m_\pi} \right) \]
\[ \left. \times \ln \left( \frac{m_\pi^2 - m_\mu^2}{2\Delta E m_\pi} \right) - \left( \frac{m_\pi^2 + m_\mu^2}{2\Delta E m_\pi} \right) \right] \]
\[ \times \frac{m_\mu^2}{m_\pi^2 - m_\mu^2} \ln \left( \frac{m_\pi^2 - m_\mu^2}{m_\pi^2 - m_\mu^2} \right) \right. \]
\[ \left. - \ln \left( \frac{m_\pi^2 - m_\mu^2}{m_\pi^2 - m_\mu^2} \right) \right] \right], \quad (2) \]

or
\[ R = \left[ 11.00 + 0.27 \ln(2\Delta E/m_\pi) \right] \times 10^{-8}, \quad (3) \]

where \( \Delta E \) and \( \Delta E/m_\pi \) are the energy intervals below the maximum energy over which electron or muon counts are being accepted. We have omitted from Eq. (3) the term which depends on \( \ln(\Delta E/m_\pi) \) since it is completely negligible. For \( 2\Delta E/m_\pi \approx 0.5 \text{ MeV} \) the radiative corrections amount to a surprisingly large decrease in the ratio of \( 14\% \).

We note that even though the rate for \( \pi/\mu \) or \( e/\mu \) + \( \nu \) contains logarithms of the ultraviolet cutoff, the ratio of rates will not depend on the cutoff if it is small enough. For \( 2\Delta E/m_\pi \approx 0.5 \text{ MeV} \), the radiative corrections amount to a surprisingly large decrease in the ratio of \( 14\% \).

The results given here do not agree with a recent "theorem" by Gatto and Ruderman. However, using a theorem due to Ruderman and Watson, a closely related statement can be made: Let \( \Gamma_P \) and \( \Gamma_A \) represent the rates for one of the pion decay channels with the leptons coupled with either \( P \) or \( A \) interaction. Then the ratio \( \Gamma_P/\Gamma_A \) including electromagnetic effects, expressed in terms of the bare masses, is equal to the ratio \( \Gamma_P/\Gamma_A \) without including electromagnetic effects. In particular, for the electron decay mode \( \Gamma_P/\Gamma_A = (m_\pi^2/b)/m_e^2 (b) \) with and without including radiative corrections.

are interested in
\[ \left( \frac{\Gamma_e/\Gamma_{\mu}}{\Gamma_e/\Gamma_{\mu}} \right) = \left[ \frac{\Gamma_e/\Gamma_{\mu}}{\Gamma_e/\Gamma_{\mu}} \right] \frac{(\Gamma_e/\Gamma_{\mu}) P}{(\Gamma_e/\Gamma_{\mu}) P}, \]

then we note that the term in brackets is equal to the ratio of the square of the bare masses of the electron and muon with or without including conventional electrodynamics. However, one should also note that \( (\Gamma_e/\Gamma_{\mu}) P \) receives a large radiative correction which to first order in \( e^2 \) is around 11% decrease.

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4Since the neutrino is not observed we have included in Eq. (2), in the calculations of the inner bremsstrahlung, the contributions from all photons having momenta consistent with the conservation laws. This means that in the integration over real photon momenta the maximum photon energy will be a function of \( x \), the cosine of the angle between electron (muon), and of the energy interval \( \Delta E \), i.e.,

\[ \omega_{\text{max}} = \left( \frac{2m_e^2}{m_\pi^2 - m_e^2} \right) (\Delta E e^2/\eta^2). \]

5If, for example, we take the cutoffs occurring in the virtual processes to be proportional to the respective masses of electron or muon, then \( R \) increases by about 1.5% over that given by Eq. (3).


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MUON K-CAPTURE COMPARED TO 
\( \beta \) DECAY FOR \( C^{12} \longrightarrow B^{12} \)

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Recent refinements of the theory of the \( V-A \) universal Fermi interaction have made it desirable to attempt a more precise experimental