Solution of the Strong $CP$ Problem by Color Exchange

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We present a new way to solve the strong $CP$ problem in models with a spontaneously broken $CP$ invariance. It is simpler than existing non-Peccei-Quinn approaches. It predicts the existence of light (i.e., weak scale) colored Higgs bosons which could be seen in colliders.

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In this Letter we propose a new way of solving the strong $CP$ problem.\textsuperscript{1} The two key ingredients of our approach are that (1) $CP$ invariance is spontaneously broken, and (2) light colored scalar fields mediate the $CP$-nonconserving interaction.

One of us (A.Z.) proposed\textsuperscript{2} recently the introduction into the standard SU(3) $\otimes$ SU(2) $\otimes$ U(1) theory of an SU(2)-singlet scalar field $\chi$, thus allowing Yukawa couplings such as $d_{u}C_{d_{u}}\chi$, $d_{u}C_{u}\chi$, and $s_{u}C_{u}\chi$. The field $\chi$ has to transform as $6^{*}$ under color in order for the couplings to be symmetric in family. This proposal was part of a program\textsuperscript{3-7} to look for new physics in the scalar sector of the standard theory.

It was also suggested\textsuperscript{2} that $\chi$ exchange could generate a $\Delta S = 2$, $CP$-nonconserving scalar interaction. A similar suggestion was made independently by Nieves\textsuperscript{8} who constructed a specific model\textsuperscript{9} at the SU(3) $\otimes$ SU(2) $\otimes$ U(1) level. We note, however, that the spontaneous breaking of $CP$ invariance at the weak scale leads to a nasty domain-wall problem. We thus propose models at the grand-unified level instead, counting on inflation to resolve the domain-wall problem.

The central point of this paper is that the color-exchange approach “naturally” solves the strong $CP$ problem. In our approach, the entire quark-mass matrix (and not only its determinant) is real at tree level. This idea was first proposed some time ago by one of us (S.B.) and Seckel,\textsuperscript{10} who were unable to implement it in a technically natural way, however. Indeed, the vital element missing in Ref. 10 is the existence of light colored scalars, a point which will be discussed in detail below.

We impose $CP$ invariance on the Lagrangean so that the Yukawa couplings are real at tree level. Thus the tree-level quark-mass matrix $M_{\mathrm{QCD}}^{\mathrm{tree}}$ is completely real. [The phase of the vacuum expectation value (VEV) of the Weinberg-Salam doublet $\phi$ is meaningless and may be chosen to be real by a global hypercharge rotation.] Note that this is a consequence of there being only one $\phi$. Were there two or more, their VEV’s could and would have $CP$-nonconserving relative phases that would show up in the tree-level quark-mass matrix.

The tree-level value of $\Theta_{\mathrm{QCD}}$ is also zero by $CP$ invariance; thus

$$\Theta_{\mathrm{tree}} = \Theta_{\mathrm{QCD}}^{\mathrm{tree}} + \arg \det M_{\mathrm{QCD}}^{\mathrm{tree}} = 0.$$  \hfill (1)

Most remarkably, as we will see, the quantum numbers of $\chi$ are such that the one-loop correction to the phase of the quark-mass matrix vanishes identically. A small two-loop correction is generated, and for a certain range of parameters, this correction may be detectable by electric-dipole-moment measurements.

$CP$ nonconservation in the $\chi$-exchange process $d + d \rightarrow \chi \rightarrow s + s$ can come either from the $\chi$ propagator or from the $\chi$-quark vertex. In the first case, there must be more than one $\chi$, or else the propagator is real by Hermiticity.

We now construct grand-unified models to illustrate these two possibilities. The moment that we mention grand unification (GU), we are faced with the hierarchy problem, of course. In line with standard practice, we will have to simply decree that certain components of a grand multiplet are light (i.e., with mass $<< M_{\mathrm{GU}}$) while other components are heavy (i.e., with mass $~ M_{\mathrm{GU}}$). Both of our models are based on SU(5) with fermions of each family in $10_{L} + 5_{L}^{*}$.

In model $A$, the Higgs bosons are in $5_{H}$, $24_{H}$, $15_{A}$, and $R_{B}^{B}$ ($A,B = 1,2,\ldots$. We decree that the SU(2) doublet in $5_{H}$ and the color-6 $\chi^{4}$ in each of $15_{A}$ are light, while the rest are all heavy. Here $R$ can be any representation containing an SU(3) $\otimes$ SU(2) $\otimes$ U(1) singlet. $CP$ invariance is broken spontaneously by the VEV of the singlet components of $R$, which can have nontrivial relative phases. This $CP$ nonconservation will be communicated to the entire Higgs sector (through couplings such as $A_{ABCD}^{15^{D}15^{D}} R^{C} R^{D}$). In particular, the propagator of the light color-6’s, $\chi^{4}$, will be $CP$ nonconserving.

[A particularly economical choice, given essentially by Nieves,\textsuperscript{8} involves taking the SU(5) singlet which transforms as $R_{H} \rightarrow - R_{H}$ under $CP$. The coupling $\frac{1}{\sigma} 15^{D} 15^{D} R + \text{H.c.}$, with $\sigma$ real and $B \neq A$, generates $CP$ nonconservation when $R_{H}$ acquires a VEV.]

SU(5) allows the Yukawa coupling

$$\mathcal{L} = f_{mn}(10_{ml}, 10_{nl}) 5_{H} s_{ml} + g_{mn}(10_{ml}, 5_{nl}^{*}) 5_{H}^{*} + h_{mn}(5_{ml}^{*}, 5_{nl}^{*}) 15_{A}^{*}.$$  \hfill (2)
Here \( m \) and \( n \) label generations.

Writing \( \Delta^2/M^2 \) for the measure of \( CP \) nonconservation in the \( \chi \) propagator (here \( \Delta^2 \) is essentially the imaginary part of \( A_{ABCD}(R^{C*}) (R^D) \), \( M^2 \) is the average mass squared of the \( \chi \), and \( h_{11} \) and \( h_{22} \) are some combinations of the \( h_{11}^I \) and \( h_{22}^I \)), we find that the experimental value of the \( \epsilon \) parameter in the \( K_L - K_S \) system requires

\[
(h_{11} h_{22} / M^2) (\Delta^2 / M^2) \sim 3 \times 10^{-15} \text{ GeV}^{-2}.
\]

(3)

We can use either the vacuum-insertion method or the bag model to estimate the matrix element \( \langle K_0 | d Q d S q_R s_R | K_0 \rangle \).

It is not unreasonable to imagine the couplings \( h_{mn}^I \) to be of the same order as the standard Higgs couplings \( g_{mn} \). Thus with \( h_{11}^I \sim g_{11} \equiv 2 \times 10^{-5} \), \( h_{22}^I \sim g_{22} \equiv 5 \times 10^{-4} \), and \( \Delta^2 / M^2 \equiv 10^{-14} \), Eq. (3) gives \( M \cong 550 \) GeV. We note that this model can accommodate “maximal” \( CP \) nonconservation, \( \Delta^2 / M^2 \sim O(1) \).

How big does \( \Theta \) come out to be in such a model? Either chirality or group theory guarantees that there is no one-loop diagram that contributes to \( \Theta \). For instance, at the SU(5) level the \( 15_H \) has to be emitted and absorbed to have \( CP \) nonconservation. The quark-mass terms always involve the \( 10_L \), but the \( 15_H \)'s do not couple to the \( 10_L \). The lowest-order graph that contributes to the \( CP \)-nonconserving part of the quark-mass matrix involves two loops as shown in Fig. 1. Crudely, it is of order

\[
\delta M_{mn} \sim \sum_{l,m} \lambda g_{il} \left( \frac{g_{lm}^* h_{11}^I h_{mn}^I}{h_{11}^I h_{22}^I} \right) \left( 3 \times 10^{-15} \text{ GeV}^{-2} \right) M^2 = (5 \times 10^{-14}) \left( \sum_{l,m} \lambda g_{il} \left( \frac{g_{lm}^* h_{11}^I h_{mn}^I}{h_{11}^I h_{22}^I} \right) \right)^2 \left( \frac{M}{300 \text{ GeV}} \right)^2.
\]

(4)

Roughly then (with the assumption that phases of diagonal components of \( M_Q \) dominate),

\[
\bar{\Theta} \equiv \Theta_{\text{QFD}} = \arg \det M_Q
\]

\[
\sim \frac{1}{64 \pi^2} \sum_{n,l,m} \lambda \left( g_{nl} \right) \left( \frac{h_{nm}^I h_{mn}^I}{h_{11}^I h_{22}^I} \right) \left( 3 \times 10^{-15} \text{ GeV}^{-2} \right) M^2 = (5 \times 10^{-14}) \left( \sum_{n,l,m} \lambda g_{nl} \left( \frac{g_{nm} h_{11}^I h_{mn}^I}{h_{11}^I h_{22}^I} \right) \right)^2 \left( \frac{M}{300 \text{ GeV}} \right)^2.
\]

(5)

In model A, replaced by \( \arg (h_{11}^I h_{22}^I) \), which measures the \( CP \) nonconservation in the \( \chi \) vertices in model B. Again the lowest-order graph that contributes to \( \Theta \) is Fig. 1. The estimate in Eq. (5) is still valid if we replace \( \Delta^2 / M^2 \) by \( \arg (h_{11}^I h_{mn}^I) \).

Let us try to put the approach suggested above in historical perspective. The various approaches to the strong \( CP \) problem are charted in Table I. Virtually all successful models based on spontaneously broken \( CP \) invariance have been based on having \( M_Q \) complex at tree level, so as to generate a sufficiently large \( \epsilon \) through the Kobayashi-Maskawa mechanism, and yet having \( \det M_Q \) real at tree level to make \( \Theta \) sufficiently small. This requires a special form for \( M_Q \). The simplest and most satisfactory of these models are those of the type discovered by Nelson.19 A simpler ap-

![FIG. 1.](image)

The lowest-order contribution to \( \Theta \) in the models described in the text comes from such two-loop diagrams.
proach would seem to be to have $M_Q$ completely real at tree level and have $\epsilon$ generated through some non-Kobayashi-Maskawa mechanism such as a $\Delta S = 2$ Higgs boson exchange. This was first suggested in Ref. 10. That paper, however, only considered the case in which the Higgs bosons $\chi$ which mediated the superweak force were color-singlet, weak doublets like the Weinberg-Salam Higgs bosons. This leads to a fa-
tal difficulty. Such $\chi$ will naturally acquire a VEV when $SU(2) \otimes U(1)$ breaks. Such a VEV must contribute, of course, to $M_Q$. And, since the VEV of the ordinary Weinberg-Salam doublet, $\phi$, and that of $\chi$ will (naturally) have a CP-nonconserving relative phase, $M_Q$ will be complex at tree level and $\Theta \gg 10^{-9}$. At-
ttempts to protect $\chi$ from acquiring a VEV by impos-
sion of symmetries are futile as shown in Ref. 10. The essence of the argument is simple: Both $\phi$ and $\chi$ couple to the quarks. Therefore, diagrams like Fig. 3 exist, and therefore, it cannot be possible to rule out terms like $X^\dagger \phi$ in the Higgs potential, which will induce $\langle X \rangle \neq 0$.

The idea that we put forward here is essentially that of Ref. 10 with the crucial difference that the $\chi$ here are taken to be colored. Obviously, the difficulty en-
countered there does not occur here: As long as color is unbroken, $\langle X \rangle = 0$ and cannot contribute to $M_Q$.

The crucial feature of this model is that all phases are drained from $M_Q$ at tree level, thus severing the link between $\Theta$ and $\epsilon$ until two-loop order.

A model which has some similarities to the present idea was proposed by Masiero, Mohapatra, and Pec-
ci. In their left-right model $\Theta_{QFD} \neq 0$ even at tree level; however, it is of order $(M_{W_L}/M_{W_R})^2$, the ratio of the squares of scales of the breaking of $SU(2)_L$ and $SU(2)_R$, which is itself, of course, a technically unna-
tural hierarchy. In common with the present mechanism their model has the features that $M_Q$ as a whole is nearly real, and that the $\Delta S = 2$, superweak, CP-
nonconserving force is mediated by color-$6$ fields. (See Table I.)

Since our mechanism requires the existence of light colored scalars, we must look at the effect on $\sin^2\theta_W$. Introduction of one light scalar in a $(6,1,-\frac{1}{2})$ of $SU(3) \otimes SU(2) \otimes U(1)$ will change $\sin^2\theta_W$ by $-0.02$. This is tolerable, but perhaps makes $\sin^2\theta_W$ a little small for comfort. With two such light $(6,1,-\frac{1}{2})$ representations as in model A (or a unified version of Nieves’s model), $\delta(\sin^2\theta_W) = -0.04$, which gives $\sin^2\theta_W$ too small. However, one can improve matters if in the $15_H$ both the $(6,1,-\frac{1}{2})$ and the $(1,3,1)$ are light (the $(3,2,\frac{1}{2})$ can cause proton decay and must be superheavy). Then $\delta(\sin^2\theta_W) = -0.0077$ for each such $15_H$.

As a first concluding remark, we emphasize that the preceding models are only meant as illustrations of a very general mechanism, the key ingredients of which are three: (1) CP a spontaneously broken symmetry; (2) a light colored scalar to mediate a superweak force; (3) a single Weinberg-Salam doublet (or some other assumption) that ensures that $\Theta_{QFD} = 0$. Obviously these requirements are not very restrictive and can be satisfied in a wide class of theories.

The second remark regards baryon- and lepton-number-nonconserving effects. As is well known, the minimal $SU(5)$ conserves $B-L$ because of a global symmetry with quantum numbers $X(10_e) = 1$, $X(5_f) = -3$, and $X(5_H) = -2$. This symmetry is preserved in our models with $X(15_H) = 6$. If desired, one can also readily break $B - L$ by a cubic Higgs coupling $\rho S_H 5_H 15_H$. This violates $B - L$ by two units and generates a neutron-antineutron oscillation matrix element, $M_{n\bar{n}}$, of order

$$M_{n\bar{n}} \sim \frac{1}{2} \langle g_{11} \rangle^2 \rho M_5^6/M_4^2 M_5^4$$

(6)

(where $M$ and $M_5$ are the masses of $X$ and the color triplet in $5_H$). It has been emphasized that with scalar fields transforming as color 6 one can construct a model with observable $n-n$ oscillation without exces-
sive proton decay. The breaking of $B - L$ also allows the appearance of neutrino Majorana masses, the mechanism being that the $\langle S_H \rangle$ induces a vacuum expectation value for the field which is a $(1, 3, 1)$ of $SU(3) \otimes SU(2) \otimes U(1)$ contained in the $15_H$. The neutrino Majorana masses are of order $p h_{mn} \times \langle S_H \rangle^2 / M^2_\text{triplet}$. If $M_\text{triplet}$ is around $10^{15}$ GeV, then $M_{\nu}$ comes out typically fairly small $M_{\nu} \approx (h_{11} / g_{11}) \langle P / (10^{15} \text{ GeV}) \rangle \times 10^{-7} \text{ eV}$. However, with a large value of $\rho$ [which is probably required by Eq. (6) to give observable $n\bar{n}$ oscillations], one could also get observable neutrino masses. On the other hand, if the $(1, 3, 1)$ of Higgs bosons is light as suggested above to get a better fit for $\sin^2 \theta_W$, then $\rho$ had better be zero (e.g., by the global symmetry), or else $M_{\nu}$ will be far too large.

The third remark is that a characteristic feature of the models described here is that $CP$ nonconservation is absent in the $D\bar{D}$ system as has been noted independently by Nieves. 8

Fourthly, the color-$6$ field, $\chi$, may be produced in pairs by a gluon in hadronic collisions. 24 Once produced, the $\chi$ may grab two quarks to form a new class of hadron ($\chi q\bar{q}$) with mass of order of hundreds of gigaelectronvolts. If $h_{33}$ is the largest coupling, as we have assumed, then these hadrons will tend to decay rapidly into the bottom channel. Abandoning our theoretical prejudice that the $\chi$ mass is weak scale, we can also speculate on the phenomenology of new hadrons of the form ($\chi\bar{d}d$), ($\chi\bar{d}d$), ($\chi\bar{u}u$), and so forth with masses in the tens of gigaelectronvolt range and whose decays into channels such as $ss\bar{u}u = (K^- K^-)$ and $sb\bar{u}u = (B^- K^-)$ provide striking signatures. 25 In this case, according to Eq. (3), the couplings $h_{11}$ and $h_{22}$ would have to be smaller by a factor of 10 than what we had stated. $\Omega$ would be smaller by a factor of $10^2$.

Finally, one might consider making such a model supersymmetric. Several potential difficulties would have to be dealt with. Since there are two light Higgs doublets, $H$ and $H'$, one would have to ensure that the phase of $\mu$ (in $\mu HH'$) was sufficiently small. At tree level this would be the guaranteed by $CP$ invariance. Also one must worry about the masses of the fermionic partner of the $\chi$ ($\tilde{\chi}$) and about gluino-quark couplings. The color-$6$ fermion, $\tilde{\chi}$, could be of use in condensing to generate the weak scale. 26 These questions are being investigated.

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