Enhanced CP Violation with $B \rightarrow KD^0(D^0)$ Modes and Extraction of the Cabibbo-Kobayashi-Maskawa Angle $\gamma$

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The Gronau-London-Wyler method extracts the Cabibbo-Kobayashi-Maskawa angle $\gamma$ by measuring $B^\pm$ decay rates involving $D^0/D^0$ mesons. CP violation can be greatly enhanced for decays to final states common to both $D^0$ and $\bar{D}^0$ that are not CP eigenstates. Large asymmetries are possible for final states $f$ such that $D^0 \rightarrow f$ is doubly Cabibbo suppressed while $\bar{D}^0 \rightarrow f$ is Cabibbo allowed. The measurement of interference effects in such modes allows the clean extraction of $\gamma$.

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One striking implication of the standard model with three families is that it can accommodate CP violation via the Kobayashi-Maskawa mechanism [1]. Intense experimental efforts are now underway in $B$ physics to test the standard model in this regard through measurements of the unitarity triangle [2]. For this program to succeed it is of crucial importance to be able to deduce each of the angles of this triangle from experiment. In this paper we will focus our attention to one of the three angles, namely the Cabibbo-Kobayashi-Maskawa (CKM) phase angle $\gamma = \arg(-V_{ub}V_{cb}V_{td}V_{cd})$. In the standard model, $\gamma$ is the relative phase between $b \rightarrow c\bar{c}s$ and $b \rightarrow u\bar{c}s$ transitions. In order to measure CP violation due to this phase, a means must be found to have these seemingly distinct final states interfere. A mechanism whereby this is possible has been proposed and extensively studied [3–8]. The basic idea is that if the $\bar{u}c$ ($\bar{c}u$) hadronize into a single $D^0$ ($\bar{D}^0$) meson, which is subsequently seen as a CP eigenstate (e.g., $K_S\pi^0$) or $K_S + n\pi$, then both processes lead to a common final state. These two channels can thus interfere quantum mechanically giving rise to, in particular, CP violating effects [3].

The Gronau-London-Wyler (GLW) method [4–7] extracts the CKM angle $\gamma$ from measurements of the branching ratios of the six processes, $B^- \rightarrow K^-\bar{D}^0$, $K^-D^0$, $K^-D^0_{CP}$, and their CP-conjugate partners. Here $D^0_{CP}$ denotes that the $D^0$ or the $\bar{D}^0$ is seen in a CP eigenstate. The two interfering amplitudes have a CP violating phase difference $\gamma$. The observation of CP violation also requires a CP even strong phase difference. This will generally be present due to final state interactions although it is not known how to calculate it reliably. Even if this phase difference is small, information about $\gamma$ may still be extracted from CP even interference effects.

The use of $D^0$ and $\bar{D}^0$ decays to common states that are not CP eigenstates was proposed several years ago [7]. In this Letter we wish to point out that among this category, $D^0$ decays which are doubly Cabibbo suppressed lead to CP violating effects that may be greatly enhanced. In addition, a number of potential experimental difficulties with the GLW method may be reduced or overcome.

In the GLW method, CP violating asymmetries tend to be small since $B^- \rightarrow K^-\bar{D}^0$ is color suppressed, whereas $B^- \rightarrow K^-D^0$ is color allowed. Moreover, when the appropriate CKM factors are taken into account, the former amplitude is typically an order of magnitude smaller than the latter. In the GLW method the interference effects are therefore expected to be limited to $O(1\%)$, which indicates the maximum possible size for CP violation via this method. To overcome this we choose instead $D^0$ modes, $f$, that are not CP eigenstates. Especially appealing are modes $f$ such that $D^0 \rightarrow f$ is doubly Cabibbo suppressed while $\bar{D}^0 \rightarrow f$ is Cabibbo allowed (e.g., $f = K^+\pi^-$, $K\pi\pi$, etc.). As a result, the two interfering amplitudes become comparable; see Fig. 1. Numerically, the ratio between these two amplitudes is crudely given by

$$\left|\frac{\mathcal{M}(B^- \rightarrow K^-D^0 \rightarrow f)}{\mathcal{M}(B^- \rightarrow K^-\bar{D}^0 \rightarrow f)}\right|^2 \approx \left|\frac{V_{ub}V_{us}^*}{V_{ub}V_{cs}^*}\right|^2 \frac{1}{a_1} \frac{1}{a_2} \frac{B(D^0 \rightarrow f)}{B(\bar{D}^0 \rightarrow f)} = \frac{0.22}{0.08} \frac{1}{0.26} \approx 0.0077 \sim 1,$$

where $\mathcal{M}$ denotes the amplitude for the given process. Here the color-suppressed amplitude ($\sim a_2$) is reduced with respect to the color-allowed one ($\sim a_1$) by the factor suggested in [9], $|a_2/a_1| = 0.26$, and the ratio of CKM elements $|V_{ub}/V_{cb}| = 0.08$ was used.

While a naive estimate for the ratio of twice Cabibbo suppressed to Cabibbo-allowed branching ratio is $B(D^0 \rightarrow f)/B(\bar{D}^0 \rightarrow f) = \lambda^4$, where $\lambda = \theta_c \approx 0.22$, the form-factor and decay constant ratios may increase the estimate somewhat. Such a ratio has been
Let us define, for a general final state followed by a strong phase difference between states, a partial rate asymmetry (PRA):

$$\frac{B(D^0 \rightarrow K^+ \pi^-)}{B(D^0 \rightarrow K^- \pi^+)} = 0.0077 \pm 0.0025 \pm 0.0025,$$

which was used in Eq. (1) for the generic ratio.

Let us denote the above branching ratios as $a(K) = B(B^- \rightarrow K^- D^0)$, $b(K) = B(B^- \rightarrow K^- \bar{D}^0)$, $c(f_i) = B(D^0 \rightarrow f_i)$, and $c(\bar{f}_i) = B(D^0 \rightarrow \bar{f}_i)$. For a given final state $f_i$, let us define $d(K, f_i) = B(B^- \rightarrow K^- f_i)$ and $\bar{d}(K, f_i) = B(B^- \rightarrow K^- \bar{f}_i)$ where the square bracket denotes that the bracketed mode originates from a $D^0/\bar{D}^0$ decay. In the standard model, $\sigma(K) = B(B^- \rightarrow K^+ D^0) = a(K)$ and $\bar{\sigma}(K) = b(K)$.

Likewise, $\sigma(\bar{f}_i) = B(D^0 \rightarrow \bar{f}_i) = c(f_i)$ and $\sigma(\bar{f}_i) = c(\bar{f}_i)$.

Equation (1) suggests that CP violating effects in the interference of two amplitudes of this type can be large. Let us define, for a general final state $f$, the CP violating partial rate asymmetry (PRA):

$$A(K, f) = \frac{d(K, f) - \bar{d}(K, f)}{d(K, f) + \bar{d}(K, f)}.$$

The largest CP violating asymmetry $A(K, f)$ in $B^\pm$ decays involving $D^0 - \bar{D}^0$ interference may occur when $f$ is a doubly Cabibbo suppressed decay mode of the $D^0$.

In the GLW method where $f$ is a CP eigenstate, the strong phase difference between $D^0 \rightarrow f$ and $\bar{D}^0 \rightarrow f$, $\delta_f = \arg[\mathcal{M}(D^0 \rightarrow f)\mathcal{M}(\bar{D}^0 \rightarrow f)]$, is to an excellent approximation 0 mod $\pi$. Therefore, the total strong phase difference involved is that of the initial $B$ decay, $\xi_K$ mod $\pi$, where $\xi_K$ is given by

$$\xi_K = \frac{1}{2} \arg[\mathcal{M}(B^- \rightarrow K^- D^0)\mathcal{M}(B^- \rightarrow K^- \bar{D}^0)]^* \mathcal{M}(B^+ \rightarrow K^+ D^0) \mathcal{M}(B^+ \rightarrow K^+ \bar{D}^0).$$

Since $A(K, f) \propto \sin(\xi_K + \delta_f)$, if $\xi_K$ should happen to be small the GLW method will produce only a small CP violating signal. In contrast, $\delta_f$ may assume different values for various non-CP eigenstates $f$. Some of these phases could be large. Indeed, some experimental evidence suggests that final state interaction effects in such $D^0$ decays can be appreciable [12]. Since several such modes are experimentally feasible, for instance $f = K^+ \pi^-, K^0 \rho^-, K^+ a_1, K^{*+} \pi^-, K_\pi \pi$, etc., it is likely that for at least some of these $\sin(\xi_K + \delta_f)$ will be large leading to a large asymmetry $A(K, f)$.

If it were feasible to determine both $a(K)$ and $b(K)$, then a single final state $f$—which may be either a CP eigenstate (as in [4,5]) or a non-CP eigenstate [7] (such as doubly Cabibbo suppressed modes)—allows the extraction of $\gamma$ (up to a fourfold discrete ambiguity) from the experimental observables $a(K)$, $b(K)$, $c(f)$, $c(\bar{f})$, $d(K, f)$, $\bar{d}(K, f)$. A potential problem with this method of extracting $\gamma$ is, though, that while $a(K)$ can be measured by conventional means outlined below, $b(K) \sim O(10^{-6})$ suffers from some serious experimental difficulties. First, if $b(K)$ is measured through the use of hadronic decays of the $\bar{D}^0$ (e.g., $\bar{D}^0 \rightarrow K^+ \pi^-$), then interference effects of $O(1)$ with the $D^0$ channel (e.g., $B^- \rightarrow K^- D^0 \rightarrow K^+ \pi^-$) will occur [see Eq. (1)]. Clearly, then, the $\bar{D}^0$ must be tagged with a decay that is distinct from any decay of the $D^0$, for instance the semileptonic decay $D^0 \rightarrow l^- \pi^- X$. This mode, however, is subject to daunting backgrounds, such as $B^- \rightarrow l^- \pi^- X$. which is $O(10^6)$ times larger. Such backgrounds may be difficult to overcome [13].

The possibility of having a variety of strong phases allows for several methods for the extraction of $\gamma$ [14]; here we discuss only one. We assume that all relevant branching ratios for $D^0$ decays are known [11].

The first method assumes that $a(K)$ is known but not $b(K)$. Indeed $a(K)$ can be determined via Cabibbo allowed modes of $D^0$ decay (e.g., $D^0 \rightarrow K^+ \pi^-$, $K^- \rho^-$. The decay chain $B^- \rightarrow K^- D^0 \rightarrow l^- \pi^- X_2$. This mode, however, is subject to daunting backgrounds, such as $B^- \rightarrow l^- \pi^- X$, which is $O(10^6)$ times larger. Such backgrounds may be difficult to overcome [13].

The second method also requires the branching ratios for at least two distinct final states $f_1$ and $f_2$ (where at least one of $f_1$, $f_2$ is not a CP eigenstate), i.e., $d(K, f_1)$, $\bar{d}(K, f_1)$, $d(K, f_2)$, and $\bar{d}(K, f_2)$. This information suffices to extract $\gamma, B(B^- \rightarrow K^- D^0)$, and the two relevant strong phase differences up to some discrete ambiguity.

To see how this works, let us write the expressions for $d(K, f_i)$ and $\bar{d}(K, f_i)$ in terms of the strong phases and $\gamma$:

$$d(K, f_i) = a(K)c(f_i) + b(K)c(\bar{f}_i) + 2\sqrt{a(K)b(K)c(f_i)c(\bar{f}_i)}\cos(\xi^K + \gamma),$$

$$\bar{d}(K, f_i) = a(K)c(f_i) + b(K)c(\bar{f}_i) + 2\sqrt{a(K)b(K)c(f_i)c(\bar{f}_i)}\cos(\xi^K - \gamma),$$

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where \( i = 1, 2 \) and \( \xi_{f_i}^K = \xi_K + \delta_{f_i} \). These four equations contain the unknowns \( \{\xi_{f_i}^K, \xi_{f_i}, b(K), \gamma\} \) which therefore can be determined up to discrete ambiguities. Adding more modes reduces, in general, the ambiguity to an overall twofold one in the sign of all the phases.

This method also illustrates the importance of \( D \) decay studies in interpreting such \( CP \) violation in \( B \) decays. The strong phases \( \xi_{f_i}^K \) relevant to Eq. (3) are related to the \( D \) decay phase shifts \( \delta_{f_i} \) via

\[
\xi_{f_i}^K - \xi_{f_i} = \delta_{f_i} - \delta_{f_i}. \tag{4}
\]

Since the separate phase shifts \( \delta_{f_i} \) on the right hand side of (4) may be determined from detailed studies of \( D \) decays or from data at \( \psi(3770) \) charm factory [14,15], this relation puts an additional constraint on the system of Eqs. (3). Indeed, if \( \delta_{f_1} \) and \( \delta_{f_1} \) are known then \( \xi_K \) may also be extracted, thereby providing information about final state interaction effects in \( B \) decays.

The discussion above may be generalized to \( B \) decays of the form \( B^+ \to K^- d^0 \) versus \( K^- d^0 \) where \( K, K^* \) or any higher kaonic resonance. Likewise, \( d^0 \) denotes \( D^0, D^{*0} \), or any higher \( D \) resonance where that excited state cascades down to a \( D^0 \) that in turn decays to final states accessible to both \( D^0 \) and \( \bar{D}^0 \) [16]. This immediate generalization is constrained to the cases where either the \( K^- \) or the \( d^0 \) spin 0, otherwise several partial waves are present. The case with multiple partial waves may still be considered, except that each of the amplitudes may have a different strong phase and so must be separated.

Let us now give a rough numerical estimate of the typical size of the asymmetry \( A(K, f) \) and the number of \( B \)'s needed to observe the effects. We shall perform the estimate for the case \( B^- \to K^- [K^+ \rho^-] \). We start with the known branching ratio \( B(B^- \to \rho^- D^0) = 1.3\% \). Multiplying this by the Cabibbo factor of \( \lambda^2 \) one obtains an estimate of \( a(K^+) = 6.6 \times 10^{-4} \). Using the CKM values and the color factor as in Eq. (1), one obtains \( b(K^+) = 6 \times 10^{-6} \). The experimental value of \( c(K^\rho^+) = 0.11 \). To estimate the value of \( c(K^\rho^-) \) let us assume that \( c(K^\rho^-) = c(K^\rho^+) \). For illustrative purposes, we compare results for \( A \) and \( N^{3\sigma} \) of the same order of magnitude as the \( B^- \to K^- [K^+ \rho^-] \) case.

In terms of the angles \( \xi_{K^+ \rho^-}^K \) and \( \gamma \), the PRA is given by

\[
A(K^+, K^+ \rho^-) = -R(K^+, K^+ \rho^-) \sin \xi_{K^+ \rho^-}^K \sin \gamma / [1 + R(K^+, K^+ \rho^-) \cos \xi_{K^+ \rho^-}^K \cos \gamma], \tag{5}
\]

where

\[
R(K^+, K^+ \rho^-) = \frac{2\sqrt{2}a(K^+)b(K^+)c(K^+ \rho^-)c(K^+ \rho^+)}{a(K^+)c(K^+ \rho^-) + b(K^+)c(K^+ \rho^+)}.
\]

For the numbers above, then, \( R = 0.99 \). To estimate the asymmetry, we need the values of the weak and strong phases. The strong phases cannot be reliably calculated and \( \gamma \) is, at present, not very well constrained experimentally. For illustrative purposes, we choose \( \cos \xi_{K^+ \rho^-}^K \cos \gamma = 0 \) so that the denominator in Eq. (5) assumes its average value between \((1 + R)\) and \((1 - R)\). Let us also take \( \sin \xi_{K^+ \rho^-}^K \sin \gamma = 1/2 \), because 1/2 is the root mean square average value of \( \sin \theta_1 \sin \theta_2 \) for randomly selected \((\theta_1, \theta_2)\). The resulting asymmetry is \( A \sim 50\% \). We now define \( N^{3\sigma} \) to be the total number of charged \( B \)'s [i.e., \( N^{3\sigma} = N(B^+) + N(B^-) \)] required to observe \( A \) to \( 3\sigma \) significance without including detector efficiencies. Thus \( N^{3\sigma} = 18/[A^2(a(K^+, K^+ \rho^-) + d(K^+, K^+ \rho^-))] \), which in this case would be \( N^{3\sigma} = 3 \times 10^7 \). Similarly, for the case of \( B^- \to K^- [K^+ \rho^-] \), \( N^{3\sigma} = 7.5 \times 10^7 \).

As a comparison, one can perform a similar estimate for the case where \( f \) is a \( CP \) eigenstate as in the GLW method. For \( f = K_S \pi^0 \), \( \sin \xi_K \sin \gamma = 1/2; \cos \xi_K \cos \gamma = 0 \), we get \( R = 0.19 \), \( A = 9.5\% \), and \( N^{3\sigma} = 15.5 \times 10^7 \). In the GLW method it is possible to combine statistics for all \( CP \) eigenstate modes. If one does not include modes with \( K_L \) this amounts to a branching fraction which is roughly \( 5\% \) of \( D^0 \) decays. Taking \( 5\% \), we find \( N^{3\sigma} = 3 \times 10^7 \), about the same as for our single mode above. In [14] similar estimates are performed for the modes \( B^- \to K^- [K^+ \rho^-] \),
provide a rough idea of the reach of such modes. Note also that another similar generalization of our method is to consider \( B \rightarrow K + n \pi + D^0(\overline{D}^0) \) decays.

Modes such as \( B^- \rightarrow D^0 \pi^- \), \( D^0 \rho^- \), \( D^0 a_1^- \) could also lead to observable CP violating effects if \( D^0 \) is seen in doubly Cabibbo suppressed modes \([7]\). \( \gamma \) can be extracted via analogous methods to the ones outlined above. The expected effects are somewhat less optimal with regards to the previously discussed modes, however.

Finally, let us comment on \( D^0 \) decay modes which are singly Cabibbo suppressed yet not CP eigenstates such as \( K^{+\pm} K^- \), \( K^{*+} K^{*0} \), \( \pi^+ \rho^- \), \( \pi^0 a_1^- \), \( \rho^- a_1^- \), etc. Since for these modes the quark content is self-conjugate \( c(f) = c(\overline{f}) \). Thus, as with the true CP eigenstate modes of the GLW method, the CP violating effects from \( B^- \rightarrow K^-[f] \) will be \( O(10\%) \) and \( N^{3\sigma} \) will be similar to that estimated above for the GLW case. The ability to generate strong phase differences with non-CP eigenstates may prove an important advantage over the GLW method, especially for small \( \xi_K \).

On the other hand, \( B^0 \rightarrow K^0 D^0 \) and \( B^0 \rightarrow K^0 \overline{D}^0 \) are color suppressed and so \( D^0(\overline{D}^0) \) decays to such singly Cabibbo suppressed modes could lead to large CP asymmetries. Indeed, such an approach, which again provides an additional strong phase difference due to \( D^0 \) decays, may significantly enhance and refine the methods discussed in \([4,6]\) where CP eigenstates are used.

In summary, the use of \( B^- \rightarrow K^- D^0 [\rightarrow f] \) and \( B^- \rightarrow K^- \overline{D}^0 [\rightarrow f] \), where \( f \) is a doubly Cabibbo suppressed mode of the \( D^0 \) and thus a Cabibbo allowed mode of the \( \overline{D}^0 \), appears promising. In many such modes the CP asymmetries are expected to be sizable. By combining information from the observed modes of the \( D^0 \), extraction of the CKM angle \( \gamma \) should be feasible. For instance, the study of two or more such modes, along with information from Cabibbo allowed modes of \( D^0 \), should enable an accurate extraction of \( \gamma \).

Various \( B \) detectors currently under construction are designed to observe mixing-induced CP violation. Such experiments should be able to determine the CKM phase \( \beta \) without any assumption concerning the strong phases. Likewise, both for the original GLW method \([5]\) and our version, \( \gamma \) is reconstructed (up to discrete ambiguities) without any assumption about the value of the strong phase. Moreover, the absence of penguin contributions in these methods allows a clean extraction of \( \gamma \), i.e., free of hadronic uncertainties. The ability to probe \( \gamma \) more incisively improves our capacity to constrain or rule out the standard model. Since these methods measure direct CP violation rather than oscillation effects, one may perform such experiments at any facility where \( B \) mesons are copiously produced. Because neither tagging nor time-dependent studies are required, such effects could be observed at even a symmetric Y(4S) factory, such as CLEO. To optimize the observation and interpretation of such effects, accurate measurements of the relevant \( D^0 \) decays are highly desirable.

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[8] M. Witherell (private communication); S. Stone, Nucl. Instrum. Methods A 333, 15 (1993); A. Snyder, BaBar notes No. 80, No. 84.
[11] We neglect \( D^0-\overline{D}^0 \) mixing and CP violation in \( D^0 \) decays, since the standard model predicts them to be very small.
[13] The signal via semileptonic \( \overline{D}^0 \) decays does have some distinguishing features which may reduce backgrounds: (1) In the \( B^- \) frame the \( K^- \) is monochromatic. (2) The signal involves two kaons, whereas the background tends to have only one. (3) The semileptonic decay of the \( D^0 \) originates from a tertiary vertex, in contrast to background.
[16] Some excited \( d^0 \) states have additional flavor specific decay modes such as \( d^0 \rightarrow D^{(*)0} \pi^- \). Such modes may therefore in principle be used to determine \( b(k) \) for \( B^- \rightarrow K^- \overline{d}^0 \) and the GLW method may then be feasible.