Determination of the Fundamental Parameters of the $K^0$-$\bar{K}^0$ System in the Energy Range 30–110 GeV

S. H. Aronson

Accelerator Department, Brookhaven National Laboratory, Upton, New York 11973

and

G. J. Bock

Enrico Fermi Institute, University of Chicago, Chicago, Illinois 60637

and

Hai-Yang Cheng and Ephraim Fischbach

Physics Department, Purdue University, West Lafayette, Indiana 47907

(Received 26 February 1982)

$K^0$ regeneration data in the energy range 30–110 GeV have been analyzed to determine the values of the $K_L$–$K_S$ mass difference, $\Delta m$, $K_S$ lifetime, $\tau_S$, and CP-nonconservation parameter $\eta_s$. We find $\Delta m = 0.482(14) \times 10^{-12}$, $\tau_S = 0.905(7) \times 10^{-16}$ s, $|\eta_s| = 2.09(2) \times 10^{-3}$, and $\tan \phi_{\eta*} = 0.709(102)$, corresponding to $\phi_{\eta*} = 35(4)^\circ$. The data suggest that these parameters may have an anomalous energy dependence.

PACS numbers: 14.40.Aq, 13.25.+m, 13.20.Eb

In this Letter we describe an analysis of data from three $K^0$ regeneration experiments to determine for the first time the fundamental parameters of the $K^0$–$\bar{K}^0$ system at Fermilab energies. The details of the work summarized here are to be published elsewhere.

In the experiments analyzed here, a $K^0$ beam impinged on a material target (hydrogen, carbon, or lead) and the $K^0\rightarrow\pi^+\pi^-$ decay distribution in the forward direction behind the targets was studied. The proper time distribution of the decays is given by

$$N_{\pi\pi}(t) = N_L \left\{ |\rho|^2 \exp(-t/\tau_S) + |\eta_{\pi\pi}|^2 \exp(-t/\tau_L) + 2|\rho| |\eta_{\pi\pi}| \exp(-t/2\tau_S - t/2\tau_L) \cos(\Delta m t + \Phi) \right\},$$

where $N_L$ is the flux of $K_L$'s, $t = 0$ at the exit face of the target, and $\Phi = \arg \rho - \arg \eta_{\pi\pi} = 0$. The

Vol. 1, p. 1971; R. Cowsik and J. McClelland, Phys. Rev. Lett. 25, 669 (1972); Ap. J. 189, 7 (1973), or heavier than about 2 GeV (see Ref. 7) and lighter than the temperature of the universe after the gravitinos decay. The last proviso is added here to guarantee that the gauge fermions have time to annihilate; Eq. (4) shows that it can be satisfied only if $\Delta E > 10^{12} \xi^{-1/2}$ GeV. This proviso can be somewhat relaxed if the gauge-fermion mass is larger than a few gigaelectronvolts. A more thorough discussion of gauge-fermion annihilation in a specific model is given by S. Dimopoulos and S. Raby, to be published.


14These temperatures are low enough that we can ignore the explicit breaking of supersymmetry by finite temperature effects, discussed by A. Das and M. Kaku, Phys. Rev. D 18, 4540 (1978); L. Girardello, M. T. Grisaru, and F. Salomonson, Nucl. Phys. B178, 331 (1981).

15This may raise the early baryon-entropy ratio above levels which can be accounted for by baryon-nonconserving processes in the very early universe. For instance, if $\Delta E = 10^{12}$ GeV so that $m_\pi = 10^{11}$ GeV, then (4) gives an entropy increase by a factor $> 10^4$, a present value of $10^{-10}$ for the baryon-entropy ratio would thus correspond to an early value of $> 10^{-6}$, which seems rather high. Of course, those would be no problem here if one were to suppose that baryon number is conserved, and that a large baryon-entropy ratio was put in at the beginning, or that $\Delta E > 10^{13}$ GeV, in which case baryon production could occur after the gravitinos decay.
coherent regeneration amplitude $\rho$ for a target of length $L$ with $N$ scatterers per unit volume is given by

$$\rho = i\pi N\lambda_5 \alpha(L/\Lambda_5) [f(0) - f(0)]/k.$$  \hfill (2)

In Eq. (2) $\Lambda_5$ is the $K^0$ mean decay length, $k$ is the $K^0$ wave number, and $f(0)$ is the forward elastic scattering amplitude of $K^0$ ($\bar{K}^0$) for the target used. The function $\alpha$ contains the effects of integrating the regeneration amplitude over a finite length target:

$$\alpha(L/\Lambda_5) = \frac{1 - \exp(-\beta L/\Lambda_5)}{\beta}.$$  \hfill (3)

In Refs. 1–3 the $\pi^+\pi^-$ decay distributions in 10-GeV kaon energy bins were fitted with Eq. (1) to determine $|\rho|$ and $\varphi_\rho$, and in Refs. 1 and 2 $N_L$ was measured with $K^-\pi^+\pi^0\mu^+\nu$ decays collected simultaneously. The parameters $\Delta m$, $\tau_5$, $\beta L$, $|\eta_{\pi^+}|$, and $\varphi_{\pi^+}$ were set equal to their accepted values$^8$ as determined at low energy.

Here we have analyzed the data, treating $\Delta m$, $\tau_5$, and $|\eta_{\pi^+}|$ as variables in the fits and letting them attain their “best” values. Because the data cannot sustain a fit to this many free parameters, we have constrained $|\rho|$ and $\varphi_\rho$ to lie near their published values.$^7$ A systematic study of the effects of these “soft” constraints is presented by Aronson et al.$^4$

The phase $\varphi_{\pi^+}$ of $\eta_{\pi^+}$ cannot, however, be extracted as directly from these data, and must be handled differently. As inspection of Eq. (1) indicates, the measured quantity is $\Phi = \varphi_{\pi^+} - \varphi_{\pi^-} = \pi/2 + \varphi_{\eta_{\pi^+}} - \varphi_{\eta_{\pi^-}}$, where $\varphi_{\eta_{\pi^+}} = \arg(\alpha(L/\Lambda_5))$ and $\varphi_{\eta_{\pi^-}} = \arg[f(0) - f(0)]$. Thus, it is necessary to supply external information on $\varphi_{\eta_{\pi^+}}$ to obtain $\varphi_{\pi^+}$. We have done this by computing $\varphi_{\eta_{\pi^+}}$ with several Regge-pole and absorption models. The results of these calculations$^8$ are consistent in the amount of energy dependence of $\varphi_{\eta_{\pi^+}}$ predicted in our energy range.$^9$ The phase results quoted here use $\varphi_{\eta_{\pi^+}}$ from the recent model of Diu and Ferraz de Camargo.$^9$

The fit to Eq. (1) was carried out in two different ways, denoted by A and B.

**Method A.**—The entire 30–110-GeV energy range was used to determine the best values of each of the parameters; these are given in Table I. If the parameters are taken to be constants, then the best values are

$$\Delta m = 0.482(14) \times 10^{10} \text{ MeV} \text{ c}^{-1}, \quad \tau_5 = 0.905(7) \times 10^{-10} \text{ s},$$

$$|\eta_{\pi^+}| = 2.09(2) \times 10^{-3}, \quad \tan \varphi_{\pi^+} = 0.709(102).$$  \hfill (4)

Note that our result for $|\eta_{\pi^+}|$ is in excellent agreement with the value $2.15(14) \times 10^{-3}$ obtained earlier by Birulev et al.$^{10}$ in the range 14–50 GeV. Note also that the results in Eq. (4) differ from the corresponding low-energy values$^6$ ($E_0 \approx 5$ GeV) by 4, 2, 9, and 3 standard deviations for $\Delta m$, $\tau_5$, $|\eta_{\pi^+}|$, and $\tan \varphi_{\pi^+}$, respectively. This has led us to investigate a possible energy dependence of the $K^0$–$\bar{K}^0$ parameters. Accordingly,

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Energy-independent fit</th>
<th>Fits of the form $x = x_0(1 + b_x \gamma^2)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10^{10}\Delta m$</td>
<td>$a = 0.482 \pm 0.014$</td>
<td>$x_0 = 0.557 \pm 0.036$</td>
</tr>
<tr>
<td>(b) &amp; $0.533 \pm 0.002$</td>
<td>$10^{10}\tau_5$</td>
<td>$0.533 \pm 0.002$</td>
</tr>
<tr>
<td></td>
<td>$c = 0.532 \pm 0.002$</td>
<td>$604/492$</td>
</tr>
<tr>
<td></td>
<td>$0.534 \pm 0.002$</td>
<td>$573/492$</td>
</tr>
<tr>
<td>$10^6</td>
<td>\eta_{\pi^+}</td>
<td>$</td>
</tr>
<tr>
<td>(10$^{-3}$) &amp; $b = 2.14 \pm 0.01$</td>
<td>$536/488$</td>
<td>$0.892 \pm 0.002$</td>
</tr>
<tr>
<td>&amp; $c = 2.07 \pm 0.01$</td>
<td>$604/492$</td>
<td>$521/484$</td>
</tr>
<tr>
<td></td>
<td>$1.069 \pm 0.036$</td>
<td>$536/488$</td>
</tr>
<tr>
<td></td>
<td>$1.061 \pm 0.040$</td>
<td>$604/492$</td>
</tr>
</tbody>
</table>

$^8$Internal fit.

$^9$External fit, with low-energy values at $E_0 \approx 5$ GeV: $\Delta m = 0.5349 \pm 0.0022$, $\tau_5 = 0.8923 \pm 0.0022$, $|\eta_{\pi^+}| = 2.274 \pm 0.022$, and $\tan \varphi_{\pi^+} = 0.986 \pm 0.055$.

$^a$As in b, except $|\eta_{\pi^+}| = 1.95 \pm 0.03$.  

---

TABLE I. Results of the direct, one-step analysis A.
we allowed each parameter in Eq. (1) to vary as
\[ x = x_0[1 + b_x \gamma^N]; \quad N = 1, 2; \quad \gamma = E_K/m_K, \]
where \( x = \Delta m, \tau_5, |\eta_{+-}|, \) or \( \tan \varphi_{+-}. \) The case \( N = 2 \) is the one given in the table; the \( N = 1 \) results\(^4\) are similar. We performed fits to \( x_0 \) and the slopes \( b_x \) first using only the data of Refs. 1–3 ("internal" fits), and then including in addition a world average value\(^5\) at \( E_K \approx 5 \) GeV ("external" fits). We see from the internal fits [entries (a) in the table] that the \( b_x \) differ from zero by 3, 2, 2, and 3 standard deviations for \( \Delta m, \tau_5, |\eta_{+-}|, \) and \( \tan \varphi_{+-}, \) respectively.\(^1\)

Method B.—The data of Refs. 1–3 were fitted by Eq. (1) to determine \( \Delta m, \tau_5, |\eta_{+-}|, \) and \( \tan \varphi_{+-} \) in each of eight energy bins (10 GeV wide) from 30 to 110 GeV. The resulting values of each parameter were then fitted to Eq. (5) to determine \( x_0 \) and \( b_x. \) This two-step procedure is perhaps easier to grasp, but has a potential drawback in that correlations among the slope parameters, if they exist, are partially obscured. Figure 1 shows an example of this analysis for \( \tan \varphi_{+-}, \) and similar graphs for the other parameters are given in Ref. 4. The results obtained by Methods A and B are in good qualitative agreement, the principal difference being that Method B gives slope parameters which are somewhat smaller in magnitude and also in statistical significance.

Notwithstanding the agreement between the results of these two different methods, the possibility remains that the apparent energy dependence of the \( K^0-\bar{K}^0 \) parameters is due to some unknown systematic effect in the data. To test for this we carried out an extensive series of auxiliary tests\(^1–4\) on the data to search for various systematic effects. No effect was found which could account for the present results. We stress that the internal fit results are completely independent of the low-energy determinations of any of the parameters, and hence of any possible systematic uncertainties arising from a comparison of high- and low-energy data. When the low-energy values are included [entries (b) and (c) in the table], slope parameters of greater statistical significance (typically 3–5 standard deviations) emerge.

Effects of the type reported here, namely a dependence of quantities determined in the \( K^0 \) rest frame on the \( K^0 \) laboratory energy, could arise from the motion of the kaons with respect to some external field or medium. A phenomenological analysis of such effects in terms of external fields is treated in detail in Ref. 4. The conclusion of this analysis is that the present results cannot be explained in terms of a conventional hypercharge, electromagnetic, or gravitational field, or from the scattering of \( K^0 \) and \( \bar{K}^0 \) from stray charges or neutrinos. This suggests that these effects, if real, may arise from some new interaction. One candidate is a tensor field of finite range, a possibility which has been considered by a number of authors.\(^1\) Such a field could at the same time resolve a number of other problems, including the missing mass of the universe and possible discrepancies in determination of neutrino oscillation parameters. These and other related questions are discussed in more detail in Ref. 4.

We wish to thank our many colleagues for their help and encouragement during the course of this work. We are particularly indebted to Bruce Winstein for many valuable suggestions. This work was performed under the auspices of the U. S. Department of Energy.

---

\(^4\) S. H. Aronson et al., to be published.
\(^5\) For reviews of \( K^0 \) regeneration and \( K^0 \) phenomenology,
The Liouville theory is quantized with use of Fock-space methods, an infinite set of charges \( L_n, n = 0, \pm 1, \ldots \), is constructed which represents the conformal algebra in two dimensions, and consequences of this algebra are discussed. It is then argued, with use of variational methods in Fock space, that the spectrum of the Liouville Hamiltonian is continuous, and that there exist energy eigenstates obeying the constraints \( L_n |E> = 0 \), \( n > 0 \).

PACS numbers: 11.30.Ef, 03.70.+k, 11.30.Na, 12.40.Hh

The (1+1)-dimensional Liouville quantum field theory,\(^1\) described by
\[
\mathcal{L} = -\frac{1}{2} \left( \partial_t \varphi \right)^2 - \frac{1}{2} \mu^2 \exp\left( 2a_{1/2} \varphi / \beta \right),
\]
appears in a manifestly covariant quantization of the relativistic string.\(^2\) In this Letter we exhibit a quantization scheme for the Liouville model which maintains the conformal symmetry of the classical theory. Within this scheme the energy spectrum and certain correlation functions can be calculated exactly.

Let us first review the classical Liouville field theory.\(^3\) Since we eventually wish to apply our results to the quantum string problem, we shall study (1) on a finite space of length \( L \): \( 0 \leq x \leq L \). For simplicity we shall also choose periodic boundary conditions appropriate to closed string theories. It is convenient to introduce dimensionless coordinates \( 2\pi x = L \sigma, 2\pi t = L \tau \), \( 0 \leq \sigma \leq 2\pi \), and to employ the Fourier series expansions\(^4\)

\[
\varphi(\sigma) = \frac{i}{2\sqrt{\pi}} \left[ a_0 - a_0^\dagger + \sum_{n \neq 0} \frac{1}{n} \left( a_n e^{-in\sigma} + b_n e^{in\sigma} \right) \right],
\]
\[
\pi(\sigma) = \dot{\varphi}(\sigma) = \frac{1}{2\sqrt{\pi}} \left[ a_0^\dagger + \sum_{n \neq 0} \left( a_n e^{-in\sigma} + b_n e^{in\sigma} \right) \right],
\]
where
\[
a_n^\dagger = a_n, \quad b_n^\dagger = b_n \quad \text{for} \ n \neq 0.
\]


\(^2\)C. Bricman et al. (Particle Data Group), Phys. Lett. 75B, 1 (1978); R. L. Kelly et al. (Particle Data Group), Rev. Mod. Phys. 52, 81 (1980).

\(^3\)We "softly" constrained each parameter \( \rho ( \varphi = | \varphi |, \varphi) \) to the value \( \rho \) by adding to \( \chi \) terms of the form \( \rho - | \varphi |^2/ \sigma_2^2 \), where \( \sigma_2 \) governs the softness of the constraint. We took \( \rho \) to be the published values (see Refs. 1-3) and \( \sigma_2 \) to be 1, 3, or 6 times the published errors. We found that for all kaon energies below 110 GeV the data in fact determine all parameters rather well, the average contribution to \( \chi^2 \) from either the \( | \varphi | \) or \( \varphi \) term being \( \leq 0.04 \).

\(^4\)All models predict a change in \( \varphi(\varphi) \) of \( \leq 2 \) for hydrogen in the energy range 30-110 GeV. By contrast, our hydrogen data show that in this range \( \varphi(\varphi) \) changes by \( (19.3 \pm 6.4)^\circ \).


\(^7\)As can be seen from the table, the four constraints \( b_1 = 0 \) lead to an increase in \( \chi^2 \) of 15, 100, and 42. This means that the data exclude the hypothesis of no energy dependence at a confidence level \( > 95\% \). Note also that in the energy-dependent fits the low-energy intercepts \( x_0 \) are in better agreement with the accepted low-energy values than are the energy-independent results in Eq. (4).


Conformally Invariant Quantization of the Liouville Theory

Thomas L. Curtright and Charles B. Thorn

Physics Department, University of Florida, Gainesville, Florida 32611

(Received 9 March 1982)

The Liouville theory is quantized with use of Fock-space methods, an infinite set of charges \( L_n, n = 0, \pm 1, \ldots \), is constructed which represents the conformal algebra in two dimensions, and consequences of this algebra are discussed. It is then argued, with use of variational methods in Fock space, that the spectrum of the Liouville Hamiltonian is continuous, and that there exist energy eigenstates obeying the constraints \( L_n |E> = 0 \), \( n > 0 \).

PACS numbers: 11.30.Ef, 03.70.+k, 11.30.Na, 12.40.Hh

The (1+1)-dimensional Liouville quantum field theory,\(^1\) described by
\[
\mathcal{L} = -\frac{1}{2} \left( \partial_t \varphi \right)^2 - \frac{1}{2} \mu^2 \exp\left( 2a_{1/2} \varphi / \beta \right),
\]
appears in a manifestly covariant quantization of the relativistic string.\(^2\) In this Letter we exhibit a quantization scheme for the Liouville model which maintains the conformal symmetry of the classical theory. Within this scheme the energy spectrum and certain correlation functions can be calculated exactly.

Let us first review the classical Liouville field theory.\(^3\) Since we eventually wish to apply our results to the quantum string problem, we shall study (1) on a finite space of length \( L \): \( 0 \leq x \leq L \). For simplicity we shall also choose periodic boundary conditions appropriate to closed string theories. It is convenient to introduce dimensionless coordinates \( 2\pi x = L \sigma, 2\pi t = L \tau \), \( 0 \leq \sigma \leq 2\pi \), and to employ the Fourier series expansions\(^4\)

\[
\varphi(\sigma) = \frac{i}{2\sqrt{\pi}} \left[ a_0 - a_0^\dagger + \sum_{n \neq 0} \frac{1}{n} \left( a_n e^{-in\sigma} + b_n e^{in\sigma} \right) \right],
\]
\[
\pi(\sigma) = \dot{\varphi}(\sigma) = \frac{1}{2\sqrt{\pi}} \left[ a_0^\dagger + \sum_{n \neq 0} \left( a_n e^{-in\sigma} + b_n e^{in\sigma} \right) \right],
\]
where
\[
a_n^\dagger = a_n, \quad b_n^\dagger = b_n \quad \text{for} \ n \neq 0.
\]

© 1982 The American Physical Society

1309