MAGNETIC MONOPOLES AND COSMOLOGY

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Received 29 September 1975
Revised manuscript received 1 February 1976

The product of the mass \( m \) and annihilation ou for magnetic monopoles is obtained through a naive calculation and is shown to be greater than about \( 10^{20} \) (normalized to the proton). One concludes that either monopoles have remarkably large masses and/or \( \omega \), or that the calculation is invalid. Possible explanations of this result are discussed.

Even since the development of quantum electrodynamics the existence of magnetic monopoles has been subject to speculation [1]. Although theoretically there is no known reason to preclude the existence of monopoles, all experimental results save one [2] have been negative and that one has not been confirmed. In this letter we report a calculation of the magnetic monopole number density in the framework of the big bang cosmology.

The basis of this calculation is the solution of the Boltzmann transport equation in evolutionary cosmological models obtained by one of us previously [3]. In ref. [3] the surviving proton-antiproton pair density was calculated as a function of time, and the result argued against the existence of bulk antimatter in our universe. A similar argument leads to an important inequality for magnetic monopoles, viz.,

\[
\frac{m}{m_p} \frac{\omega_v}{(\omega_v)_p} \gtrsim (10^{14} - 10^{22})
\]  

(1)

where \( m \) is the monopole mass, \( \omega_v \) is the low energy annihilation transition probability for monopoles, and the subscript \( p \) refers to the same quantities for protons. The range of numerical values in eq. (1) reflects the upper limits of several different experiments to determine the monopole number density as summarized in ref. [4].

In the early epoch of an expanding world model the radiation energy density and particle-antiparticle pair energy density dominate the total energy density (for those particles of species \( A \) such that \( kT \gg m_A c^2 \)). In the radiation era the Hubble "constant" \( H(t) \), temperature \( T(t) \), and net particle density \( \Delta N(t) = N(t) - \bar{N}(t) \) (\( N \) and \( \bar{N} \) are the particle and antiparticle number densities, respectively) as functions of the time \( t \) (sec) after creation are given by

\[
H = 1/2t, \quad T = T'(t'/t')^{-1/2}, \quad \Delta N = \Delta N'(t'/t')^{-3/2}.
\]  

(2a,b,c)

Different cosmological theories give somewhat different (but similar) values for \( T' \) and \( \Delta N' \). Alpher, Follin and Herman [5] give \( T' = 1.5 \times 10^{10} \) K for \( t' = 1 \) sec. The evaluation of \( \Delta N' \) follows from the general relation

\[
\Delta N = \Delta N'_p (T/T')^3 (\Delta N/\Delta N'_p)_{\text{now}}
\]  

(3)

and \( \Delta N'_p = N'_p = 10^{-7} \) cm\(^{-3} \), \( T = 2.7 \) K give \( \Delta N'_p = 1.7 \times 10^{22} \) cm\(^{-3} \). The proper inclusion of interactions between particles and antiparticles will alter only slightly the numerical values of these parameters [e.g. 6].

Using eqs. (2) the Boltzmann transport equation describing nonequilibrium processes in an expanding universe is solved and applied for the case in which (a) the initial particle-antiparticle pair density is much larger than the equilibrium pair density, and (b) the transition probability \( \omega_v \) for the annihilation process is constant. The solution in terms of initial values denoted by subscript 0 is [3]

\[
\Delta N = \Delta N_0 (t/t_0)^{-3/2}
\]  

(4a)

\[
N = \frac{N_0(t/t_0)^{-3/2}}{(N_0/\Delta N_0) + (1 - N_0/\Delta N_0) \exp[-\alpha(1 - (t_0/t)^{1/2})]}
\]  

(4b)

\[
\bar{N} = \frac{\bar{N}_0(t/t_0)^{-3/2}}{(1 + \bar{N}_0/\Delta N_0) \exp[\alpha(1 - (t_0/t)^{1/2})] - \bar{N}_0/\Delta N_0}
\]  

(4c)

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\[ \alpha \equiv 2 \Delta N_0 \sigma v t_0. \quad (4d) \]

For \( \alpha \ll 1 \) and \( t \gg t_0 \) eq. (4b) becomes

\[ N = \frac{N_0 (T/T_0)^3}{1 + 2 \Delta N_0 \sigma v t_0} \tag{5} \]

through use of eq. (2b).

Now we find a time \( t_0 \) such that at \( t_0 \) the temperature and density of the universe approximate the conditions under which eqs. (4) were derived. For temperature \( T \) such that \( kT \gg mc^2 \) the number density of particle–antiparticle pairs in equilibrium with the radiation field is roughly

\[ N_\infty = a T^4 / kT \approx 10^{32} t^{-3/2} \text{ cm}^{-3} \]

where \( a \) is the Stefan–Boltzmann constant. For temperatures \( T \) such that \( kT \ll mc^2 \) the equilibrium particle–antiparticle pair density is

\[ N_\infty = \frac{\frac{3}{4} \pi^3}{\frac{3}{2}} \left( \frac{mc}{\pi \hbar} \right)^3 \left( \frac{2}{\Phi} \right)^{3/2} e^{-\Phi} \]

\[ = 1.4 \times 10^{40} \left( m/m_p \right)^{3/2} T_{13}^{3/2} \times \exp(-1.09 m/m_p T_{13}) \text{ cm}^{-3} \]

where \( \Phi = mc^2/kT \) and \( T_{13} = T \times 10^{-13} \text{ K} \). The time \( t_\ast \) at which \( N_\infty = N_\infty \) is

\[ t_\ast \approx 10^{-5} \left( m/m_p \right)^2 \text{ sec}. \quad (6) \]

For \( t < t_\ast \) the number of particle–antiparticle pairs is much larger than for \( t > t_\ast \) due to the steep temperature dependence of the pair density \( N_\infty \). Consequently, taking \( t_0 = t_\ast \) and

\[ T_0 = T_\ast t_\ast^{-1/2} \text{ K} \quad (7a) \]

\[ N_0 = \tilde{N}_0 = 10^{32} t_\ast^{-3/2} \text{ cm}^{-3} \quad (7b) \]

satisfies the conditions under which eqs. (4) were derived.

If \( \alpha \ll 1 \) eqs. (5), (6) and (7) lead to the condition

\[ \frac{m}{m_p} \frac{\sigma v}{(\sigma v)_p} \frac{N}{N_p} \approx 10^{-10}, \quad (8) \]

where \( N_p = 10^{-7} \text{ cm}^{-3} \) and \( (\sigma v)_p = 2 \times 10^{-15} \text{ cm}^3/\text{sec} \) [7] have been used. If \( \alpha \gg 1 \) eqs. (4) imply \( N \gg \tilde{N} \) so \( \Delta N \approx N \). Hence from eq. (4d)

\[ \frac{m}{m_p} \frac{\sigma v}{(\sigma v)_p} \frac{N}{N_p} \geq 10^{-10}. \quad (9) \]

Eqs. (8) and (9) are valid for any stable particle. Normalization to the proton parameters is a matter of convenience only. Specializing to magnetic monopoles one uses the experimental upper limits [4] of \( N/N_p \leq (10^{-24} - 10^{-32}) \) in eqs. (8) and (9) to get finally

\[ \frac{m}{m_p} \frac{\sigma v}{(\sigma v)_p} \geq (10^{14} - 10^{22}). \quad (10) \]

Note that the range of values in eq. (10) reflects the upper limits of several different experiments summarized in ref. [4], not the possible error in any one experiment.

Of course, if monopoles do not exist eq. (10) is meaningless. Assuming the existence of monopoles one should examine the admittedly naive assumptions under which eq. (10) was derived. Assumptions (a) and (b) above should be reasonably valid approximations: (a) because of the very steep temperature dependence of the number density for \( t > t_\ast \), and (b) because if the distribution functions for pole and antipole fall off rapidly at high energies then only the low energy \( \sigma v \) will make a contribution.

Much more important are the following: (1) At time \( t \sim t_\ast \), the average separation between poles and antipoles is of order \( 10^{-14} m/m_p \text{ cm} \). If \( m \approx m_p \) this separation is smaller than the separation between nucleons in nuclei! The electromagnetic interactions between poles and/or antipoles are classically \( V(r) \sim g^2/r \sim 10^4 e^2/r \) which are enormous at these small distances. Further, one expects non-negligible interactions between the monopoles and other particles in the cosmic soup. All these intense interactions could be expected to affect the distribution function for the monopoles. It is not known whether these effects could account for the 20 orders of magnitude of eq. (10).

(2) If a pole-antipole bound state exists [8] with very large binding energy then eq. (10) is not valid. The poles and antipoles could be preferentially paired for \( t \gg t_\ast \), and the density of these bound pairs would evolve as \( T^3 \) with subsequent annihilation at an unknown rate. However, the binding energy would have to be an appreciable fraction of the rest mass energy of a monopole (say \( |E_b| > mc^2/10 \) or the bound pair would be photodisintegrated for \( t_\ast < t < 10 t_\ast \)).

In conclusion eq. (10) admits several interpretations.

1) Monopoles exist but have remarkably larger masses and/or \( \sigma v \). 2) Monopoles exist but the intense inter-
actions between poles and other poles and particles in the high density early universe affect the annihilation of poles and antipoles for $t > t_*$ in such a manner that eq. (10) is reduced by 20 orders of magnitude. 3) Monopoles exist but possess a pole-antipole bound state with binding energy on the order of the monopole rest mass energy. 4) Magnetic monopoles do not exist.

The authors believe the last two possibilities are the most likely.

References