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Neutral $B$ mesons ($b\bar{q}$, with $q = d, s$ for $B_d^0, B_s^0$) oscillate from particle to antiparticle due to flavor-changing weak interactions. The probability density $P_\pm (P_-)$ for a $B_q^0$ meson produced at proper time $t = 0$ to decay as a $\bar{B}_q^0$ ($B_q^0$) at time $t$ is given by

$$P_\pm(t) = \frac{\Gamma_q}{2} e^{-\Gamma_q t} [1 \pm \cos(\Delta m_q t)]$$

where $\Delta m_q$ is the mass difference between the two mass eigenstates $B_{q, H}^0$ and $B_{q, L}^0$ [1], and $\Gamma_q$ is the decay width.
which is assumed to be equal for the two mass eigenstates. The mass differences $\Delta m_j$ and $\Delta m_i$ can be used to determine the fundamental parameters $|V_{td}|$ and $|V_{ts}|$, respectively, of the Cabibbo-Kobayashi-Maskawa (CKM) matrix [2], which relates the quark mass eigenstates to the flavor eigenstates. This determination, however, has large theoretical uncertainties. A measurement of $\Delta m_i$ combined with $\Delta m_j = 0.505 \pm 0.005 \text{ ps}^{-1}$ [3,4] would determine the ratio $|V_{td}/V_{ts}|$ with a significantly smaller theoretical uncertainty, contributing to a stringent test of the unitarity of the CKM matrix. Earlier attempts to measure $\Delta m_j$ have yielded a lower limit: $\Delta m_j > 14.5 \text{ ps}^{-1}$ [3,5] at the 95% confidence level (C.L.). Recently the D0 Collaboration reported $17 \text{ ps}^{-1} < \Delta m_{{\cal B}_s} < 21 \text{ ps}^{-1}$ at 90% C.L. [6] using a large sample of semileptonic $B_{{\cal S}}$ [7] decays.

In this Letter we report a measurement of $\Delta m_{{\cal S}}$ using data from 1 fb$^{-1}$ of $p\bar{p}$ collisions at $\sqrt{s} = 1.96 \text{ TeV}$ collected by the CDF II detector at the Fermilab Tevatron. We begin by reconstructing $B_{{\cal D}}$ decays in hadronic ($B_{{\cal D}}^0 \to D_{{\cal S}}^\pm \pi^\mp$, $D_{{\cal S}}^0 \pi^+\pi^-\pi^0$) and semileptonic ($B_{{\cal D}}^0 \to D_{{\cal S}}^{(*) (+)}\ell^-(\ell^-\nu\ell^+\mu\ell)$) decay modes using charged particles only [8]. Using the method of maximum likelihood, we extract the value of $\Delta m_{{\cal S}}$ from the probability density functions (PDFs) that describe the measured time development of $B_{{\cal D}}$ mesons that decay with the same or opposite flavor as their flavor at production. The proper decay time for each $B_{{\cal D}}$ is calculated from the measured distance between the production and decay points, the measured momentum, and the $B_{{\cal D}}$ mass $m_{{B_S}} = 5.3696 \text{ GeV}/c^2$ [3]. The $B_{{\cal D}}$ flavor ($b$ or $\bar{b}$) at decay is determined unambiguously by the charges of the decay products.

To identify the flavor of the $B_{{\cal D}}$ at production, we use characteristics of $b$ quark production and fragmentation in $p\bar{p}$ collisions. At the Tevatron, the dominant $b$ quark production mechanisms produce $b\bar{b}$ pairs. The $b$ and $\bar{b}$ are expected to fragment independently into hadrons. In a simple model of fragmentation, a $b$ quark becomes a $B_{{\cal D}}^0$ meson when some of the energy of the $b$ quark is used to produce an $s\bar{s}$ quark pair. The $b$ and the $\bar{s}$ bind to form a $B_{{\cal D}}^0$. The remaining $s$ quark may form a $K^-$. Similarly, a $\bar{b}$ that becomes a $B_{{\cal D}}^0$ is accompanied by a $K^+$. One of the two techniques used to identify the production flavor of the $B_{{\cal D}}$ is based on the charge of these kaons (same-side tag). The second technique uses the charge of the lepton from semileptonic decays or a momentum-weighted charge of the decay products of the second $b$ hadron produced in the collision (opposite-side tag).

The hadronic and semileptonic decay modes are complementary. Because of the large branching ratio, the semileptonic decays provide a tenfold advantage in signal rate at the cost of significantly worsened decay-time resolution due to the unmeasured $\nu$ momentum. Semileptonic decays dominate the sensitivity to oscillations at lower values of $\Delta m_{{\cal S}}$. The fully reconstructed hadronic $B_{{\cal D}}$ decays have superior decay-time resolution, and our large sample of these decays is the unique feature that makes CDF sensitive to much larger values of $\Delta m_{{\cal S}}$ than other experiments.

The CDF II detector [9] consists of a magnetic spectrometer surrounded by electromagnetic and hadronic calorimeters and muon detectors [10]. The key features for this measurement include precision vertex determination provided by the seven-layer double-sided inner silicon strip detector [11,12] supplemented with a single-sided layer of silicon [13] mounted directly on the beam pipe at an average radius of 1.5 cm. The 96-layer outer drift chamber [14] is used for both precision tracking and $dE/dx$ particle identification. Time-of-flight (TOF) counters [15] located just outside the drift chamber are used to identify low momentum charged kaons.

Charm and bottom hadrons are selected using a three-level trigger system that exploits the kinematics of production and decay, and the long lifetimes of $D$ and $B$ mesons. A crucial component of the trigger system for this measurement is the Silicon Vertex Trigger [16], which selects events that contain $B_{{\cal D}}^0 \to D_{{\cal S}}^\pm \pi^\mp$ and $D_{{\cal S}}^0 \pi^+\pi^-\pi^0$ decays. The trigger configuration used to collect the heavy flavor data sample is described in [17].

To reconstruct $B_{{\cal D}}^0$ candidates, we first select $D_{{\cal S}}^\pm$ candidates. We use $D_{{\cal S}}^\pm \to \phi\pi^\mp$, $K^*(892)^0K^\mp$, and $\pi^+\pi^-\pi^0$, with $\phi \to K^+K^-$ and $K^*0 \to K^+\pi^0$; we require that $\phi$ and $K^*0$ candidates be consistent with the known masses and widths [3] of these two resonances. These $D_{{\cal S}}^\pm$ candidates are combined with one or three additional charged particles to form $D_{{\cal S}}^\pm \pi^\pm$, $D_{{\cal S}}^0 \pi^-\pi^+\pi^0$ candidates. The $D_{{\cal S}}^\pm$ and other decay products of a $B_{{\cal D}}^0$ candidate are constrained to originate from a common vertex in three dimensions. For the $K^*(892)^0K^+$ final state, we remove candidates that are consistent with the decay $D_{{\cal S}}^\pm \to K^-\pi^+\pi^+$.

We use a likelihood technique to identify muons [18] and electrons [19].

Backgrounds are suppressed by imposing a requirement on the minimum transverse momentum $p_T$ [20] of the $B_{{\cal D}}^0$ and by requiring that the $B_{{\cal D}}^0$ and $D_{{\cal S}}^\pm$ decay vertices are displaced significantly from the $p\bar{p}$ collision position. We find signals of 3600 hadronic $B_{{\cal D}}$ decays and 37,000 semileptonic $B_{{\cal D}}$ decays.

For the hadronic decays, the invariant mass distribution (see Fig. 1) has a signal centered close to $m_{{B_S}} = 5.3696 \text{ GeV}/c^2$ with a width of 14 to 20 MeV/c$^2$, depending on the decay mode. Candidates with masses greater than 5.5 GeV/c$^2$ are used to construct PDFs for combinatorial background. To remove contributions from $B_{{\cal D}}^0 \to D_{{\cal S}}^0 \pi^-\pi^0$, $B_{{\cal D}}^0 \to D_{{\cal S}}^0 \rho^-$, and semileptonic and other partially reconstructed decays, we require the mass of the decay candidates to be greater than 5.3 GeV/c$^2$. For semileptonic decays we take into account several background contributions, including $B$ meson decays to two charm mesons and real $D_{{\cal S}}^0$ mesons associated with a false lepton.

The decay time in the $B_{{\cal D}}$ rest frame is $\tau = \kappa [L_T m_{{B_S}}/p_T]$, where $L_T$ is the displacement of the $B_{{\cal D}}$. 

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decay vertex with respect to the primary vertex projected onto the \( B_s \) transverse momentum vector. The factor \( \kappa \) corrects for missing momentum in the semileptonic decays (\( \kappa = 1 \) for hadronic decays). To improve the decay-time resolution, we use event-by-event primary-vertex position measurements when computing the \( B_s \) vertex displacement. The signal decay-time distribution is modeled with

\[
P(t_i, \sigma_i) = e(t_i) \int \Gamma x e^{-\Gamma x} G(t' - t_i, \sigma_i) dt',
\]

where \( t_i \) is the measured decay time of the \( i \)th candidate, \( \Gamma_x \) is the \( B_s \) decay width, \( G(x - \mu, \sigma) \) is a Gaussian distribution of the random variable \( x \) with mean \( \mu \) and width \( \sigma \), and \( \sigma_i \) is the estimated candidate decay-time resolution. The decay-time efficiency function \( e(t) \) describes trigger and selection biases on the decay-time distribution and is determined from Monte Carlo simulation. For semileptonic decays, the \( \kappa \) distribution is determined from Monte Carlo simulation and is convoluted with the signal decay-time distribution. The missing transverse momentum from unreconstructed particles in the semileptonic decays is an important contribution to the decay-time resolution. To reduce this contribution and make optimal use of the semileptonic decays, we determine the \( \kappa \) distribution as a function of the invariant mass of the \( D_s \ell \) pair, \( m_{D_s\ell} \). The rms width of the \( \kappa \) distribution is 3% (20%) for \( m_{D_s\ell} = 5.2 \text{ GeV}/c^2 \) (3.0 \text{ GeV}/c^2).

We estimate the decay-time resolution \( \sigma_i \) for each candidate using the measured track parameters and their estimated uncertainties. We calibrate this estimate using a large sample of prompt \( D^+ \) mesons [21], which we combine with one or three charged particles from the primary vertex to mimic signal topologies. For hadronic decays, the average decay-time resolution is 87 fs, which corresponds to one-fifth of an oscillation period at the lower limit on \( \Delta m_s \) (14.5 ps \(^{-1} \)). For semileptonic decays, the decay-time resolution is worse due to decay topology and the missing momentum of unreconstructed decay products. For example, at \( t = 0 \), \( \sigma_i = 100 \text{ fs} \) (200 fs) for \( m_{D_s\ell} = 5.2 \text{ GeV}/c^2 \) (3.0 \text{ GeV}/c^2) and increases to \( \sigma_i = 115 \text{ fs} \) (380 fs) at \( t = 1.5 \text{ ps} \).

The flavor of the \( B_s \) at production is determined using both opposite-side and same-side flavor tagging techniques. The effectiveness \( Q = e D^2 \) of these techniques is quantified with an efficiency \( e \), the fraction of signal candidates with a flavor tag, and a dilution \( D = 1 - 2w \), where \( w \) is the probability that the tag is incorrect.

Opposite-side tags infer the production flavor of the \( B_s \) from the decay products of the \( b \) hadron produced from the other \( b \) quark in the event. We use lepton (\( e \) and \( \mu \)) charge and jet charge as tags, building on techniques developed for a CDF run I measurement of \( \Delta m_d \) [22]. If both lepton and jet-charge tags are present, we use the lepton tag, which has a higher average dilution.

The dilution of opposite-side flavor tags is expected to be independent of the type of \( B \) meson that produces the hadronic or semileptonic decay. The dilution is measured in data using large samples of \( B^- \), which do not change flavor, and \( B^0 \), which can be used after accounting for their well-known oscillation frequency. The combined opposite-side tag effectiveness is \( Q = 1.5\% \pm 0.1\% \), where the uncertainty is dominated by the statistics of the control samples.

Same-side flavor tags [23] are based on the charges of associated particles produced in the fragmentation of the \( b \) quark that produces the reconstructed \( B_s \). In the simplest picture of fragmentation, a \( \pi^+ (\pi^-) \) accompanies the formation of a \( B^- (B^+) \), a \( \pi^- (\pi^+) \) accompanies a \( B^0 (\bar{B}^0) \), and a \( K^- (K^+) \) accompanies a \( B^0 (\bar{B}^0) \). In run I, CDF established this method of production flavor identification in measurements of \( \Delta m_d \) [24] and the CP symmetry violating parameter \( \sin(2\beta) \) [25]. In this analysis, we use \( dE/dx \) [19] and TOF information in a combined particle identification likelihood to identify the kaons associated with \( B_s \) production. Tracks close in phase space to the \( B_s \) candidate are considered as same-side kaon tag candidates, and the track with the largest kaon likelihood is selected as the tagging track.

The performance of the same-side kaon tag for \( B^0_s \) is expected to be different than for \( B^- \) and \( \bar{B}^0 \). We predict the dilution using simulated data samples generated with the PYTHIA Monte Carlo program [26]. Control samples of \( B^- \) and \( \bar{B}^0 \) are used to validate the predictions of the simulation. The effectiveness of this flavor tag increases with the \( p_T \) of the \( \bar{B}^0 \); we find \( Q = 3.5\% \) (4.0\%) in the hadronic (semileptonic) decay sample. The fractional uncertainty on \( Q \) is approximately 25\%. This uncertainty is dominated by the differences between data and simulation for kaons found close in phase space to the \( B^0_s \) [27] and for the performance of the same-side kaon tag when applied to \( B^- \).

If both a same-side tag and an opposite-side tag are present, we combine the information from both tags assuming they are uncorrelated. The addition of the same-side kaon tag increases the effective sample statistics by more than a factor of 3.

We use an unbinned maximum likelihood fit to search for \( B_s \) oscillations. The likelihood combines mass, decay-

FIG. 1. The invariant mass distributions for \( B^0_s \to D^-_s \pi^+ \pi^- \) (left panel) and \( D^+_s \pi^- \pi^+ \pi^- \) (right panel).
time, decay-time resolution, and flavor tagging information for each candidate and includes terms for signal and each type of background. The fit is done in three stages. First, a combined mass and decay-time fit is performed to separate signal from background and to fix mass and decay-time models. Combined fits for $B_s$ mass (Fig. 1) and decay width in hadronic samples and for decay width in the semileptonic samples yield measurements consistent with established values [3]. Second, flavor asymmetries are measured for background components. The third step is a fit for $B^+_s$ - $B^o_s$ oscillations; the mass and decay-time models and background asymmetries are fixed from the previous two stages.

The signal PDF has the general form:

$$ S_\pm(t_i, \sigma_i, D_i) = \varepsilon(t_i) \int_{\frac{t_i}{2}}^{T} e^{-\Gamma t'}[1 \pm \mathcal{A} D_i \cos(\Delta m t')] \times G(t_i - t', \sigma_i) dt', $$

where $D_i$ is the $i$th candidate dilution, and $t_i$, $\sigma_i$, $G$, and $\varepsilon(t)$ have been defined previously. Following the method described in [28], we fit for the oscillation amplitude $\mathcal{A}$ while fixing $\Delta m_j$ to a probe value. When all detector effects ($D_i$, $\sigma_j$) are calibrated, the oscillation amplitude is expected to be consistent with $\mathcal{A} = 1$ when the probe value is the true oscillation frequency, and consistent with $\mathcal{A} = 0$ when the probe value is far from the true oscillation frequency. Figure 2 (upper panel) shows the fitted value of the amplitude as a function of the oscillation frequency. The sensitivity of the measurement is defined by the maximum value of $\Delta m_j$ where $\mathcal{A} = 1$ is excluded at 95% C.L. if the measured value of $\mathcal{A}$ were zero. Our sensitivity is 25.8 ps$^{-1}$ and exceeds the combined sensitivity of all previous experiments [3]. At $\Delta m_j = 17.3$ ps$^{-1}$, the observed amplitude $\mathcal{A} = 1.03 \pm 0.28$ (stat) is consistent with unity, indicating that the data are compatible with $B^+_s$ - $B^o_s$ oscillations with that frequency, while the amplitude is inconsistent with zero: $\mathcal{A} / \sigma_\mathcal{A} = 3.7$, where $\sigma_\mathcal{A}$ is the uncertainty on $\mathcal{A}$. The negative amplitudes measured at frequencies slightly below and slightly above the peak frequency are expected and are due to the finite range in signal decay time that is imposed by the trigger and selection criteria. The systematic uncertainty on $\mathcal{A}$ is mainly due to uncertainties on $\sigma_j$ and $D_j$. Since the effect of these uncertainties on $\mathcal{A}$ and $\sigma_\mathcal{A}$ are unimportant for $\Delta m_j$, the ratio $\mathcal{A} / \sigma_\mathcal{A}$ has negligible systematic uncertainty.

The significance of the potential signal is evaluated from $\Lambda = \log \left[ \mathcal{L}_{\mathcal{A}=0} / \mathcal{L}_{\mathcal{A}=1}(\Delta m_j) \right]$, which is the logarithm of the ratio of likelihoods for the hypothesis of oscillations ($\mathcal{A} = 1$) at the probe value and the hypothesis that $\mathcal{A} = 0$, which is equivalent to random production flavor tags. Figure 2 (lower panel) shows $\Lambda$ as a function of $\Delta m_j$. Separate curves are shown for the semileptonic data alone (dash-dotted line), the hadronic data alone (dotted line), and the combined data (solid line). A minimal value of $\Lambda = -6.75$ is observed at $\Delta m_j = 17.3$ ps$^{-1}$. The significance of the signal is quantified by the probability that randomly tagged data would produce a value of $\Lambda$ lower than $-6.75$ at any value of $\Delta m_j$. We repeat the fit 50 000 times with random tagging decisions, and we find this probability is 0.2%.

Under the hypothesis that the signal is due to $B^+_s$ - $B^o_s$ oscillations, we fix $\mathcal{A} = 1$ and fit for the oscillation frequency. We find $\Delta m_j = 17.31^{+0.33}_{-0.18}$ (stat) $\pm 0.07$ (syst) ps$^{-1}$ and the range $17.01$ ps$^{-1} < \Delta m_j < 17.84$ ps$^{-1}$ (16.96 ps$^{-1} < \Delta m_j < 17.91$ ps$^{-1}$) at 90% (95%) C.L. All systematic uncertainties affecting $\mathcal{A}$ are unimportant for $\Delta m_j$. The only non-negligible systematic uncertainty on $\Delta m_j$ is from the uncertainty on the absolute scale of the decay-time measurement. Contributions to this uncertainty include biases in the primary-vertex reconstruction due to the presence of the opposite-side $b$ hadron, uncertainties in the silicon-detector alignment, and biases in track fitting. The measured $B^+_s$ - $B^o_s$ oscillation frequency is used to derive the ratio $|V_{ud}/V_{us}| = \xi \sqrt{\frac{m_{ud}}{m_{us}}}$. As inputs we use $m_{ud}/m_{us} = 0.98390$ [29] with negligible uncertainty, $\Delta m_d = 0.505 \pm 0.005$ ps$^{-1}$ [3], and $\xi = 1.21^{+0.047}_{-0.035}$ [30]. We find $|V_{ud}/V_{us}| = 0.208 \pm 0.001$ (exp) $\pm 0.008$ (theor).

In conclusion, we present the first precise measurement of $\Delta m_j$. The value of $\Delta m_j$ is consistent with standard model expectations [31] and with previous bounds. Our

\begin{figure}
\centering
\includegraphics[width=0.8\textwidth]{CDF_run_II}
\caption{(Upper panel) The measured amplitude values and uncertainties versus the $B^+_s$ - $B^o_s$ oscillation frequency $\Delta m_j$. Shown in light gray and dark gray are the 95% one-sided confidence level bands for statistical uncertainties only and including systematic uncertainties, respectively. (Lower panel) The logarithm of the ratio of likelihoods for amplitude equal to zero and amplitude equal to one, $\Lambda = \log \left[ \mathcal{L}_{\mathcal{A}=0} / \mathcal{L}_{\mathcal{A}=1}(\Delta m_j) \right]$, versus the oscillation frequency. The dashed horizontal line indicates the value of $\Lambda$ that corresponds to a probability of 1% in the case of randomly tagged data.}
\end{figure}
measured value of $\Delta m_s$ allows us to determine $|V_{td}/V_{ts}|$
with unprecedented precision and can be used to improve
constraints on the unitarity of the CKM matrix and on
scenarios involving new physics.

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$h = c = 1$ and report $\Delta m_s = m_{\mu e} - m_{\mu e}$ in inverse
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[7] The symbol $B_s$ refers to the combination of $\bar{B}_s^0$ and $B_s^0$
decays.
[8] References to a particular process imply that the charge
conjugate process is included as well.