Use of a Transmission Polarimeter for a Nonmonochromatic Photon Beam

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The longitudinal polarization of photons in the range 1-10 MeV can be determined by observation of the asymmetry in the rate of transmission of the photons through a block of magnetized iron on reversal of the polarity of the magnetization [1, 2]. In this energy range the dominant photon interaction with matter is Compton scattering. The Compton scattering cross-section can be written

$$\sigma = \sigma_0 + P_\gamma P_{e^-}^{\rm Fe} \sigma_1, \tag{1}$$

where σ_0 is the unpolarized (Klein-Nishina) cross-section, P_γ is the net longitudinal polarization of the photons, $P_{e^-}^{\rm Fe}$ is the net longitudinal polarization of the atomic electrons (naively $\pm 2/26$ for saturated iron, but more accurately determined to be $\pm 2.06/26 = \pm 0.0792$), and σ_1 is the polarized cross-section [1]. Figure 1 illustrates the energy dependence of the cross sections σ_0 and σ_1.

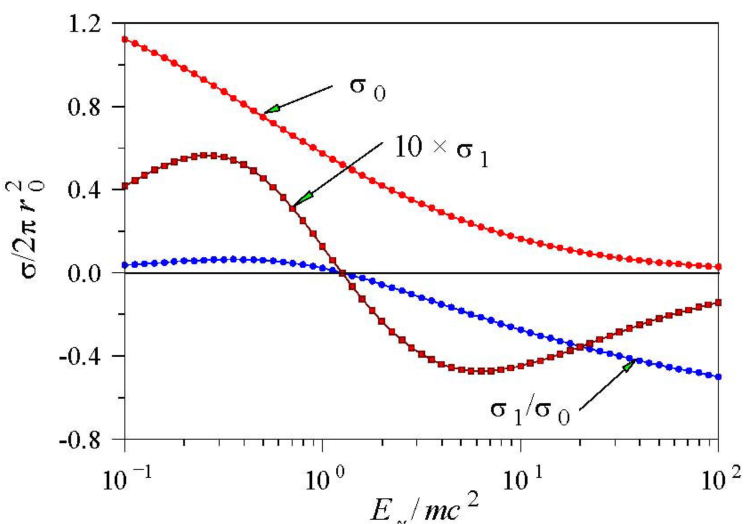

Figure 1: The total cross sections σ_0 and σ_1 for Compton scattering of longitudinally polarized photons of energy E_γ off unpolarized and longitudinally polarized electrons, respectively. r_0 is the classical electron radius.

If the apparatus is such that any photon which scatters in the iron is not detected, then the probability $T^\pm$(L,E) of transmission of a photon of energy E and longitudinal polarization P_γ through a block of iron of length L and longitudinal polarization $\pm P_{e^-}^{\rm Fe}$ can be written as

$$T^\pm(L,E) = e^{-n_{e^-}^{\rm Fe} L \sigma^\pm(E)} = e^{-n_{e^-}^{\rm Fe} L \sigma_0} e^{\pm n_{e^-}^{\rm Fe} L P_{e^-}^{\rm Fe} P_\gamma \sigma_1}, \tag{2}$$

where $n_{e^-}^{\rm Fe}$ is the number density of atoms in iron, and the $+(-)$ in $T^\pm$ applies if the electron spin in the iron is anti-parallel (parallel) to the direction of the incident photons. In case of a beam of photons with energy spectrum $N_\gamma(E)$ and longitudinal polarization that depends on energy according to $P_\gamma(E)$, the transmission is

$$T^\pm(L) = \int T^\pm(L,E) N_\gamma(E)\, dE = \int e^{-n_{e^-}^{\rm Fe} L\sigma_0(E)} e^{\pm n_{e^-}^{\rm Fe} L P_{e^-}^{\rm Fe} P_\gamma(E)\sigma_1(E)} N_\gamma(E)\, dE. \tag{3}$$

The transmission asymmetry is

$$\begin{aligned}
\delta &= \frac{T^+(L) - T^-(L)}{T^+(L) + T^-(L)} = \frac{\int e^{-n_{e^-}^{\rm Fe} L\sigma_0(E)} \left(e^{n_{e^-}^{\rm Fe} L P_{e^-}^{\rm Fe} P_\gamma(E)\sigma_1(E)} - e^{-n_{e^-}^{\rm Fe} L P_{e^-}^{\rm Fe} P_\gamma(E)\sigma_1(E)} \right) N_\gamma(E)\, dE}{\int e^{-n_{e^-}^{\rm Fe} L\sigma_0(E)} \left(e^{n_{e^-}^{\rm Fe} L P_{e^-}^{\rm Fe} P_\gamma(E)\sigma_1(E)} + e^{-n_{e^-}^{\rm Fe} L P_{e^-}^{\rm Fe} P_\gamma(E)\sigma_1(E)} \right) N_\gamma(E)\, dE} \\
&\approx n_{e^-}^{\rm Fe} L P_{e^-}^{\rm Fe} \frac{\int e^{-n_{e^-}^{\rm Fe} L\sigma_0(E)} P_\gamma(E)\sigma_1(E) N_\gamma(E)\, dE}{\int e^{-n_{e^-}^{\rm Fe} L\sigma_0(E)} N_\gamma(E)\, dE} \\
&= n_{e^-}^{\rm Fe} L P_{e^-}^{\rm Fe} \frac{\int e^{-n_{e^-}^{\rm Fe} L\sigma_0(E)} \sigma_1(E) N_\gamma(E)\, dE}{\int e^{-n_{e^-}^{\rm Fe} L\sigma_0(E)} N_\gamma(E)\, dE} \frac{\int e^{-n_{e^-}^{\rm Fe} L\sigma_0(E)} P_\gamma(E)\sigma_1(E) N_\gamma(E)\, dE}{\int e^{-n_{e^-}^{\rm Fe} L\sigma_0(E)} \sigma_1(E) N_\gamma(E)\, dE} \\
&\equiv A_\gamma P_{e^-}^{\rm Fe} \langle P_\gamma \rangle\,,
\end{aligned} \tag{4}$$

where the approximation holds when $n_{e^-}^{\rm Fe} L P_{e^-}^{\rm Fe} P_\gamma(E)\sigma_1(E)$ is small compared to 1 (as in usual practice), the analyzing power A_γ is

$$A_\gamma = n_{e^-}^{\rm Fe} L \frac{\int e^{-n_{e^-}^{\rm Fe} L\sigma_0(E)} \sigma_1(E) N_\gamma(E)\, dE}{\int e^{-n_{e^-}^{\rm Fe} L\sigma_0(E)} N_\gamma(E)\, dE}\,, \tag{5}$$

and the cross-section-weighted average polarization of the beam is

$$\langle P_\gamma \rangle = \frac{\int P_\gamma(E) e^{-n_{e^-}^{\rm Fe} L\sigma_0(E)} \sigma_1(E) N_\gamma(E)\, dE}{\int e^{-n_{e^-}^{\rm Fe} L\sigma_0(E)} \sigma_1(E) N_\gamma(E)\, dE}\,. \tag{6}$$

The weighting factor $e^{-n_{e^-}^{\rm Fe} L\sigma_0(E)}\sigma_1(E)$ is shown in Fig. 2 for $L = 15$ cm of iron, and has the approximate form $0.001(E-2)$ for $E > 2$ MeV.

Example: Suppose the photon beam has a nearly uniform energy distribution between 0 and 8 MeV, and the longitudinal polarization varies from -1 at $E = 0$ to $+1$ at $E = 8$ MeV, which approximates the conditions of SLAC experiment E166 [3]. Then, $N_\gamma(E)$ is constant and $P_\gamma = E/4 - 1$ for $0 < E < 8$ MeV, so that the average polarization according to eq. (6) is

$$\langle P_\gamma \rangle = \frac{\int_2^8 (E/4 - 1)(E-2)\, dE}{\int_2^8 (E-2)\, dE} = \frac{1}{2}. \tag{7}$$

References

[1] S.B. Gunst and L.A. Page, *Compton Scattering of 2.62-MeV Gamma Rays by Polarized Electrons*, Phys. Rev. **92**, 970 (1953),
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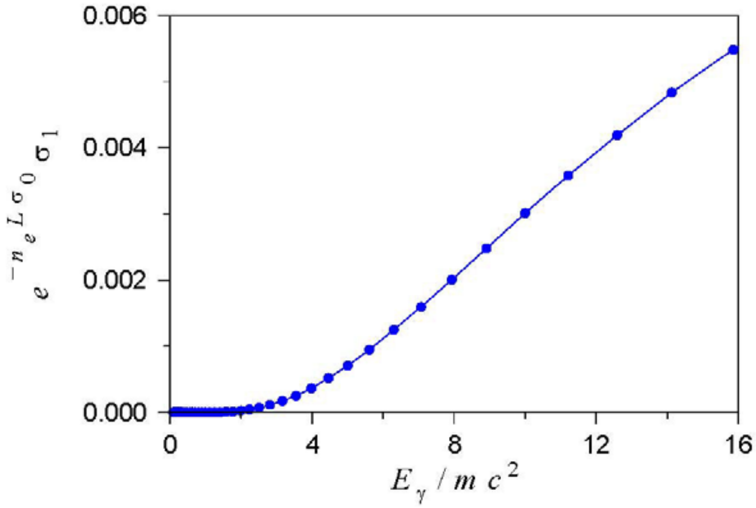

Figure 2: The polarization weighting factor $e^{-n_{e^-}^{\rm Fe} L\sigma_0(E)}\sigma_1(E)$ for $L = 15\,$cm of iron.

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