Bunch-Timing Measurement in the Muon Cooling Experiment Via $TM_{0,1,0}$ RF Accelerating Cavities

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Studies of the $TE_{0,1,1}$ and $TM_{2,1,0}$ deflection cavities show a very marginal effect.

Consider an accelerating cavity phased by 90°, so (small) energy gain is a linear function of time within the bunch.

Consider a cylindrical $\mathrm{TM}_{0,1,0}$ cavity of length b, radius a and peak field $E_0 = 40$ MV/m for $\omega = 2\pi 800$ MHz.

Dispersion relation:

$$\frac{\omega}{c} = \frac{2.405}{a}, \qquad \text{so} \qquad d = 2a = \frac{2.405}{\pi}\lambda = 0.766\lambda = 28.725 \text{ cm.}$$

Energy gain:

$$\begin{aligned}
\Delta U &= e\int E_z dz = e\beta_z c\int_{t_{\min}}^{t_{\max}} E_z dt \\
&= e\beta_z c E_0 \int_{t_{\min}}^{t_{\max}} J_0\left(\frac{2.405r}{a}\right)\sin\omega t\, dt \\
&\approx e\beta_z c E_0 \int_{t_{\min}}^{t_{\max}} \sin\omega t\, dt = \frac{\beta_z c e E_0}{\omega}\left(\cos\omega t_{\min} - \cos\omega t_{\max}\right) \\
&\approx -\frac{\beta_z c e E_0}{\omega}\frac{2\omega z_0}{\beta_z c}\sin\frac{\omega b}{2\beta_z c} = -2\sin\frac{\omega b}{2\beta_z c} e E_0 z_0 \\
&= -2eE_0\sin\frac{\omega b}{2\beta_z c}\beta_z c\Delta t,
\end{aligned}$$

$U\Delta U = c^2 P\Delta P \Rightarrow$ relative momentum change:

$$\begin{aligned}
\frac{\Delta P}{P} &= \frac{2eE_0\beta_z c\Delta t}{\beta c P}\sin\frac{\omega b}{2\beta_z c} \\
&= \frac{2\cdot 40\ [\mathrm{Mv/m}]\cdot 3\times 10^{-4}\ [\mathrm{m/ps}]\cdot \Delta t\ [\mathrm{ps}]}{165\ [\mathrm{MeV}/c]\cdot c} = 0.00014\left[\frac{\Delta t}{1\ \mathrm{ps}}\right],
\end{aligned}$$

using $b = \lambda\beta_z/3 = 10.5$ cm and $P = 165$ MeV/c.

8-Cell Cavity

Consider an 8-cell RF cavity of same design as for cooling FOFO.

We wish to resolve 8 ps $= 0.2\sigma_t$.

Then the momentum gain is $\Delta P/P = 8 \cdot 8 \cdot 0.00014 = 0.009$.

This is of same order as desired momentum resolution.

Straggling

$$\begin{aligned}\sigma_{P,\text{straggling}} &\approx \frac{\gamma r_e m_e c^2}{\beta c}\sqrt{2\pi N_0 \frac{Z}{A} s \left(1 - \frac{\beta^2}{2}\right)} \\ &\approx 0.11\ [\text{MeV}/c]\sqrt{\left[\frac{s}{1\ \text{g/cm}^2}\right]},\end{aligned}$$

Require $\sigma_{P,\text{straggling}} < \sigma_{P,D} = 0.005P = 0.82P$ MeV/c for $P =$ 165 MeV/c.

$$\Rightarrow s < \left(\frac{\sigma_{P,D}}{0.11}\right)^2 = \left(\frac{0.82}{0.11}\right)^2 = 56\ [\text{g/cm}^2].$$