A High-Energy High-Luminosity $\mu^+\mu^-$ Collider

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Abstract
We present a candidate design for a high-energy high-luminosity $\mu^+\mu^-$ collider, with $E_y = 4$ TeV, $L = 10^{35}$ cm\(^{-2}\)s\(^{-1}\), using only existing technology. The design uses a rapidly-cycling medium-energy proton synchrotron, which produces proton beam pulses which are focused onto two $\pi$-producing targets, with two $\pi$-decay transport lines producing $\mu$'s and $\mu$'s. The $\mu$'s are collected, rf-rotated, cooled and compressed into a recirculating linac for acceleration, and then transferred into a storage ring collider. The keys to high luminosity are maximal $\mu$ collection and cooling; innovations with these goals are included.

1. INTRODUCTION

Lepton(e$^+$$-$$e^-$) colliders have the valuable property of producing simple, single-particle interactions, and this property is essential in the exploration of new particle states. However, extension of e$^+$$-$$e^-$ colliders to multi-TeV energies is severely performance-constrained by beamstrahlung and cost-constrained because two full energy linacs are required.\(^1\) However, muons (heavy electrons) have negligible beamstrahlung and can be accelerated and stored in rings. The lifetimes of $\mu$'s are that they decay, with a lifetime of $2.2\times10^6$ s/m, and that they are created through decay into a diffuse phase space. However at 2 TeV that lifetime is 0.044s, sufficient for storage-ring collisions, and the phase space can be compressed and cooled. The possibility of muon colliders has been introduced by Skrinsky et al.,\(^2\) Neuffer\(^3\), and others. More recently, several mini-workshops have greatly increased the level of discussion.\(^4\) In this paper we extend these discussions, introducing improvements, particularly in $\mu$-source and cooling, and obtain high luminosity in a high energy collider. Table I shows parameters for the candidate design, which is displayed graphically in fig. 1. The scenario includes a high-intensity $\mu$-source, $\mu$-cooling, and acceleration to storage in a collider. The collider cycle is repeated at 10 Hz.

2. MUON PRODUCTION

The $\mu$-source driver is a high-intensity rapid-cycling synchrotron at KAON\(^1\) proposal parameters(30 GeV, 10 Hz) producing two bunches of $3 \times 10^{14}$ protons, which are extracted into separate lines for positives and negatives. (Separate lines permit use of higher-acceptance, zero-dispersion transports.) Each bunch collides into a 0.1m thick metal target, producing large numbers of $\pi$'s ($\sim 1$ $\pi$ interacting p) over a broad energy and angular range ($E_{\pi} = 0-4$ GeV, $p_\pi < 0.5$ GeV/c). The target is followed by Li lenses, which collect the $\pi$'s into a large-aperture ($r = 0.15m$, $B = 4T$) FODO Quad transport with a 0.8m period, designed to accept a large energy

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
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<tbody>
<tr>
<td>Energy per beam</td>
<td>$E_y$</td>
<td>4 TeV</td>
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<tr>
<td>Luminosity</td>
<td>$L$</td>
<td>$10^{35}$ cm$^{-2}$s$^{-1}$</td>
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<th>Source Parameters</th>
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<td>Protons/pulse</td>
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<tr>
<td>Pulse rate</td>
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<tr>
<td>$\mu$-production acceptance</td>
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<td>$\mu$-survival allowance</td>
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<table>
<thead>
<tr>
<th>Collider Parameters</th>
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<tbody>
<tr>
<td>Number of $\mu$ /bunch</td>
</tr>
<tr>
<td>Number of bunches</td>
</tr>
<tr>
<td>Storage turns</td>
</tr>
<tr>
<td>Normalized emittance</td>
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<tr>
<td>$\mu$-beam emittance</td>
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<tr>
<td>Interaction focus</td>
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<tr>
<td>Beam size at interaction $\sigma = (e_b)^{1/2}$</td>
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![Figure 1. A $\mu^+\mu^-$ Collider System.](image)

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width (2±1 GeV) and have a large transverse acceptance (\(\rho_\perp < 0.4\) GeV). This array is sufficiently long (≈300 m) to assure \(\pi \to \mu\) decay, plus debunching, in which the energy-dependent particle speeds spread the beam longitudinally to a full width of 6 m, while reducing the local momentum spread. This is followed by a nonlinear rf system (3 harmonics are sufficient) which flattens the momentum spread. The \(\mu\)-beam is then matched into a beam cooling system.

A Monte Carlo program has simulated the muon production and cooling. The generation of \(\pi\)'s in the target is calculated using a thermodynamic model or the Wang distribution.\(^6\) \(\pi\)'s are tracked through decay to \(\mu\)'s, \(\rho\)-E rotation, into the cooling system. We obtain ∼0.15 captured \(\mu\)'s per initial proton, with \(e_\mu = 0.01\) m-rad, an rms bunch length of 3 m, and energy width of 0.2 GeV with an average energy of 1.4 GeV. The large \(\mu\) capture efficiency (0.15/\(\mu\)) is a result of the use of a high acceptance transport, with rf rotation, followed by beam cooling.

### 3. BEAM COOLING

For collider intensities, the phase-space volume must be reduced by beam-cooling and the beam size compressed, within the \(\mu\) lifetime. Much of the needed compression is obtained through adiabatic damping in acceleration from GeV-scale \(\mu\) collection to TeV-scale collisions. Beam cooling is obtained by "ionization cooling" of muons ("\(\mu\)-cooling"), in which beam transverse and longitudinal energy losses in passing through a material medium are followed by coherent reacceleration, resulting in beam phase-space cooling.\(^2\) (Ionization cooling is not practical for protons and electrons because of nuclear scattering ('p') and bremsstrahlung ('e') effects, but is for \(\mu\)'s and the necessary energy losses are easily obtained within the \(\mu\) lifetime.) In this section we present the equations for \(\mu\)-cooling, use these to deduce optimal cooling conditions, and generate a practical cooling scenario.

The equation for transverse cooling is:

\[
\frac{d\epsilon_{\mu}}{ds} = \frac{d\epsilon_{\mu}}{dE_{\mu}} + \frac{\beta_\mu}{E_{\mu}} \frac{0.01s}{2 L_\mu} \frac{1}{E_{\mu}}
\]

(with energies in GeV), where \(e_\mu\) is the normalized emittance, \(\beta_\mu\) is the betatron function at the absorber, \(d\epsilon/ds\) is the energy loss, and \(L_\mu\) is the material radiation length. The first term in this equation is the coherent cooling term and the second term is heating due to multiple scattering. This heating term is minimized if \(\beta_\mu\) is small (strong focusing) and \(L_\mu\) is large (a low-Z absorber).

The equation for energy cooling is:

\[
\frac{d(\Delta E^2)}{ds} = -2 \frac{dE}{d\epsilon_{\mu}} \frac{\partial E}{\partial \epsilon_{\mu}} \frac{d(\Delta E^2)}{ds}
\]

Energy-cooling requires that \(\partial dE/\partial s/\partial \epsilon > 0\). The energy loss function, \(dE/ds\), is rapidly decreasing with energy for \(E_\mu < 0.2\) GeV (and therefore heating), but is slightly increasing (cooling) for \(E_\mu > 0.3\) GeV. This natural cooling is ineffective; but \(dE/\partial s\) can be increased by placing a transverse variation in absorber density or a wedge absorber where position is energy-dependent. (This variation is used in two modes: a weak variation to balance cooling rates, or a thick wedge to transfer phase space.) The sum of cooling rates is invariant:

\[
\frac{1}{E_{\text{cool}}} + \frac{1}{E_{\text{cool}}} + \frac{1}{E_{\text{cool,AB}}} = \text{Constant} = \frac{2}{E_\mu},
\]

where \(E_{\text{cool}}\) is the total energy loss needed to obtain an \(e\)-folding of cooling and \(E_\mu\) is the \(\mu\) energy.

In the long-\(\rho\)-pathlength Gaussian-distribution limit, the heating term or energy straggling term is given by:

\[
\frac{d\Delta E^2}{ds} = 4\pi r^2 \rho e^{-2} N_A Z \gamma (1-\rho^2/2)
\]

where \(N_A\) is Avogadro's number and \(\rho\) is the density. Since this increases as \(\gamma^2\), and the cooling system size scales as \(\gamma\), cooling at low energies is desired.

To obtain energy cooling and to minimize energy straggling, we require cooling at low relativistic energies \((E_\mu < 300\text{MeV})\). For optimum transverse cooling, the ideal absorber is itself a strong focusing lens which maintains small beam size over extended lengths, and a low-Z material. In this design, we use Be \((Z=4)\) or Li \((Z=3)\) current-carrying rods, where the high current provides strong radial focusing. For Be, \(Z=4, A=9\), \(dE/dx = 3\text{MeV/cm, } \rho = 1.85\text{gm/cm}^2\).

The beam cooling system reduces transverse emittances by more than two orders of magnitude (from 0.01 to \(3\times10^{-3}\) m-rad), and reduces longitudinal emittance by more than an order of magnitude. This cooling is obtained in a series of cooling cells, with the initial cells reducing the energy toward the cooling optimum of 300 MeV. A typical cooling cell consists of a focusing cooling rod (\(0.7\text{m long}\) which reduces the central energy by \(-200\text{MeV}\), followed by \(\sim 200\text{MeV}\) in a \(20-40\text{m at 10-5\text{MeV/m}}\), with optical matching sections (\(40-50\text{m total cell length})\). (See Fig. 2.) Small angle bends introduce a dispersion (position-dependence on energy), and the rod (predominantly Be) has a density gradient. Bends are also used to provide path length dependence on momentum, in order to compress the bunch lengths. The cell parameters are adjusted to optimal transverse and longitudinal cooling rates, and cooling by a 6-D factor of \(-4\) is obtained in each cell. 15-20 such cells (\(\sim 800\) m) are needed in the complete machine.

From equation 1, we find a limit to transverse cooling when multiple scattering balances cooling, at \(e_\mu = 10^2\beta_\mu\) for Be. The value of \(\beta_\mu\) in the Be rod is limited by the peak focusing field at \(\beta_\mu \sim 0.01\text{ m}, \) obtaining \(e_\mu = 10^4\text{m-rad}\). This is a factor of \(-3\) above the emittance goal of Table 1. The additional factor can be obtained by cooling more than necessary longitudinally and exchanging phase-space with transverse dimensions in a thick wedge absorber.

In the present scenario, we cool only with ionization cool-
ling in conducting Be (or Li) rods, along with phase space exchange, and that this is sufficient for high luminosity. However, other techniques (such as ionization cooling in focussing transports or rings, or using plasma lenses, or high-frequency "optical" stochastic cooling 5) may permit improvements.

4. ACCELERATION AND COLLISIONS

Following cooling and initial bunch compression to \( \sim 1\,\text{mm bunch length} \), the beams are accelerated to full energy (2 TeV). A full-energy linac would work, but it would be costly and does not use our ability to recirculate \( \mu \)'s. A recirculating linac (RLA) like CEBAF\(^1\) can accelerate to full energy in 10-20 recirculations, using only 200-100 GeV of linac, but requiring 20-40 return arcs. The \( \mu \)-bunches would be compressed on each of the return arcs, to a length of 0.003m at full energy. A cascade of RLAs (i.e., 1-10, 10-100 and 100-2000 GeV), with rf frequency increasing as bunch length decreases, may be used. Rapid-cycling synchrotrons are also possible. (A low-cost scenario requiring only 20 GeV of rf, using an injector and three RLA stages with rapid-cycling in the last stage, has been developed.) The cooling and acceleration cycle is timed so that less than \(-\frac{1}{2}\) of the initial \( \mu \) decay. (In Table 1, we allow a factor of 4 in total losses.)

After acceleration, the \( \mu \) and \( \mu \) bunches are injected into the 2-GeV superconducting storage ring, with collisions in one or two low-\( \beta \) interaction areas. The beam size at collision is \( r \approx (\sigma_{\phi} \sigma_{\theta}/\gamma)^{1/2} \), similar to hadron collider values. The bunches circulate for -300B turns before decay, where \( B \) is the mean bending field in \( T \). (This is 150B luminosity turns, a factor of two smaller because both beams decay.) The design is restricted by \( \mu \) decay within the rings (\( \mu \rightarrow e v \)), which produces 1/3-energy electrons which radiate and travel to the inside of the ring dipoles. This energy could be intercepted by a liner inside the magnets, or specially designed C-dipoles could be used and the electrons intercepted in an external absorber. The ring may be asynchronous to avoid fast instabilities. These design constraints may limit \( B \); we have chosen \( B = 4T \) (600E turns). \( \mu \)-decays in the interaction areas will also provide some background levels in detectors. The limitations in detector design are being studied.

5. COMMENTS AND CONCLUSIONS

The 4 TeV energy is set as a benchmark goal for the high-energy frontier. The \( \mu \)-\( \mu \) collider concept naturally increases in luminosity with energy, with one factor of \( E \) from emission adiabatic damping. If the injector size/cost is allowed to increase as \( E \), bunch intensities each increase as \( E \), so that \( L \) becomes proportional to at least \( E^2 \). (Smaller \( \beta \) should also be possible through longitudinal adiabatic damping, adding some further enhancement.) This scaling should follow up to the 40 TeV scale, beyond which synchrotron radiation becomes important. Lower energy machines (100+ GeV Higgs, top factories) are also possible, but require specific physics motivation.

In order to initiate practical development, some experiments are needed. An initial cooling experiment using a Be rod within a low-intensity \( \mu \)-beam is being developed. Another important experiment will explore and optimize \( \mu \) production and collection, developing the high-acceptance lines introduced here.

We have presented a candidate design for a high-energy high-luminosity \( \mu \)-\( \mu \) collider, using practical components and concepts within existing technical capabilities, and with requirements within the scope of existing facilities. The critical features of the scenario (\( \mu \)-collection and decay, phase space compression and cooling) have been modeled with Monte Carlo simulations, as well as by analytical methods. This scenario is unoptimized and will be greatly changed and improved before implementation. However, we believe that the improvements reported here have transformed the \( \mu \)-\( \mu \) collider concept into a practical and attractive possibility.

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References


Figure 2. A \( \mu \)-cooling cell.